Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation xxx (2015) xxx–xxx
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original articles

Evolution strategies for computing periodic orbits

A. Abad*, A. Elipe

Grupo de Mecánica Espacial - IUMA, University of Zaragoza, Spain

Received 8 February 2010; received in revised form 14 April 2011; accepted 17 May 2014

Abstract

An evolution strategy algorithm belonging to the general field of genetic algorithms is developed to detect periodic orbits in dynamical problems. The algorithm is applied to the problem of motion of a particle under the gravitational field of a solid circular wire.

Keywords: Periodic orbits; Evolution strategy algorithms; Motion around a solid ring

1. Introduction

An autonomous dynamical system, $\dot{\boldsymbol{x}} = F(\boldsymbol{x})$, has a periodic orbit if there is a vector $\boldsymbol{x}_0$ and a scalar T such that $\|\boldsymbol{x}(T;\boldsymbol{x}_0) - \boldsymbol{x}_0\| = 0$. Therefore, the problem of finding periodic orbits may be considered from a new view point, that is, to find the zeros of a non-negative $f : \mathcal{D} \subset \mathbb{R}^n \to \mathbb{R}^+$, or equivalently, since the function is non-negative, to find the absolute minima of the function f in the domain $\mathcal{D}$.

Modern techniques of evolutionary computation, like genetic algorithms (GA) or evolution strategies (ES), inspired on biological processes, are currently used in different scientific areas to solve problems that involve optimization of functions [1]. As we just said, we may consider the problem of finding periodic orbits as a problem of optimization, and hence this kind of methods may be applied to the determination of periodic orbits.

In this paper, we begin with a general description of the most usual features of an Evolution Strategy Method, that is, its definition, how to mutate the different items inside their influence domain, how to abandon an item and select a new one, and how to stop the procedure.

Since the problem of finding periodic orbits has no unique solution (let us remind that periodic orbits can be dense in the phase space), in order to avoid accumulation of solutions around a particular point while abandoning other regions with solutions, we make a modification to the ES algorithm and as an example, we apply it to a particular function which has several lines of zeros.

Finally, as illustration of our method, we consider the motion of a particle under the gravitational field of a solid circular wire. This problem has been studied by several authors and, in particular, by Broucke and Elipe [2], who obtained

* Corresponding author.
E-mail addresses: abad@unizar.es (A. Abad), elipe@unizar.es (A. Elipe).

http://dx.doi.org/10.1016/j.matcom.2014.05.014

Please cite this article in press as: A. Abad, A. Elipe. Evolution strategies for computing periodic orbits, Math. Comput. Simul. (2015), http://dx.doi.org/10.1016/j.matcom.2014.05.014

a map of periodic orbits for this problem. With the ES-algorithm here presented we recover the orbits obtained in the above quoted reference [2]. Let us mention that in this work we satisfy ourselves with the obtaining of periodic orbits and we neither compute the stability nor make a classification of the families, or bifurcations, which was done in [2].

We would like to emphasize that the method proposed here is only one of the methods in use, and it is good in some cases but not in all cases. Indeed, there are many of them, and researchers use one or another depending on their "taste", or on the physical characteristics of the problem considered. In principle, this method does not require a previous knowledge of the dynamics (although the more we know about the problem the better the method works); moreover the method here proposed is not intended for obtaining the whole set of periodic orbits, but only a set of them which may be used as starters for other continuation methods like the one of Deprit and Henrard [3] in which one of the major difficulties is the finding of an initial periodic orbit to continue the family.

2. Evolution strategies

An evolution strategy is an algorithm to minimize (or maximize) an objective function $f : \mathcal{D} \subset \mathbb{R}^n \to \mathbb{R}$. A detailed review about evolution strategies may be found in [1]. We write here only a short description of these methods.

A classical $(\mu/\rho + \lambda)$-ES begins with a population of μ random items $\boldsymbol{y} = (\boldsymbol{x}, \boldsymbol{\sigma})$ where $\boldsymbol{x} \in \mathcal{D} \subset \mathbb{R}^n$ is a point of the n-dimensional domain and $\boldsymbol{\sigma} \in \mathbb{R}^m$ is an m-dimensional strategy parameter vector which controls the evolution of each point (the most usual dimension of vector $\boldsymbol{\sigma}$ is either 1 or n).

By evaluating the objective function f at each given point, we may define the first generation as

$$\mathbb{G}_0 = (\boldsymbol{x}_i^0, \boldsymbol{\sigma}_i^0, f(\boldsymbol{x}_i^0)); \quad i = 1, \ldots, \mu;$$

therefore, a *generation* is a matrix $\mathbb{G} \in \mathcal{M}(n + m + 1, \mu)$.

Each point $\boldsymbol{x}$ may suffer a *mutation*. A typical mutation consists of adding a vector $\boldsymbol{z}$ to the point we want to mutate in the form $\tilde{\boldsymbol{x}} = \boldsymbol{x} + \boldsymbol{z} \in \mathcal{D}$, with $\boldsymbol{z} = (\sigma_1 y_1, \ldots, \sigma_n y_n)$, where the parameters σ_k are elements (repeated or not) of the parameter vector $\boldsymbol{\sigma}$, and y_k are random numbers obtained from normal distributions $\mathcal{N}_k(0, 1)$, that is, $\boldsymbol{z} = (\sigma_1 y_1, \ldots, \sigma_n y_n) \in (\sigma_1 \mathcal{N}_1(0, 1), \ldots, \sigma_n \mathcal{N}_n(0, 1))$.

The mutation $\boldsymbol{z}$ belongs to an n-dimensional hypercube, but the properties of the normal distribution concentrate on the mutated elements in an n-dimensional ellipsoid centered at $\boldsymbol{x}$ with more density of points near the center (see Fig. 1). If we use a one-dimensional vector parameter, then $\sigma_k = \sigma, \forall k$, and the ellipsoid is just an n-dimensional sphere of radius σ.

To build a new generation, we need to create the *offsprings*, and there are several ways in doing so. Essentially, an offspring is a mutation of a *recombination* of ρ parents (chosen at random), with ρ a predetermined fixed number between one and n. There are many strategies of recombining the parents; one of the most usual is to define the recombination $\boldsymbol{x}^\star$ of ρ parents as

$$\boldsymbol{x}^\star = \frac{1}{\rho} \sum_{i=1}^{\rho} \boldsymbol{x}_i.$$

N.B. When $\rho = 1$, there is only one parent, and the recombination is a clone of the parent. In our work, we shall take $\rho = 1$, hence we do not mention other strategies.

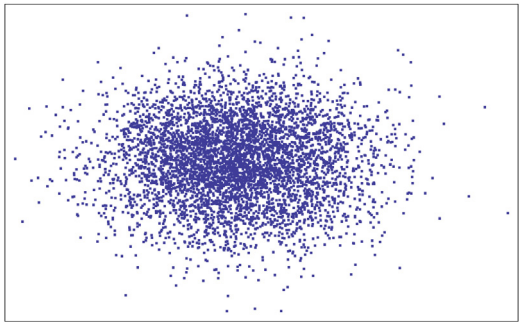

Fig. 1. Points belonging to a normal distribution.

Please cite this article in press as: A. Abad, A. Elipe. Evolution strategies for computing periodic orbits, Math. Comput. Simul. (2015), http://dx.doi.org/10.1016/j.matcom.2014.05.014