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Numerical simulation of fluid–structure interaction problems with hyperelastic models: A monolithic approach

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Abstract

In this paper, we consider a monolithic approach to handle coupled fluid–structure interaction problems with different hyperelastic models in an all-at-once manner. We apply Newton's method in the outer iteration dealing with nonlinearities of the coupled system. We discuss preconditioned Krylov sub-space, algebraic multigrid and algebraic multilevel methods for solving the linearized algebraic equations. Finally, we compare the results of the monolithic approach with those of the corresponding partitioned approach that was studied in our previous work.

Keywords: Fluid–structure interaction; Hyperelastic models; Monolithic approach; Algebraic multigrid method; Algebraic multilevel method

1. Introduction

Parallel to the development of the partitioned approach for the fluid–structure interaction (FSI) simulation (see, e.g., [22,7,51,36,28]), the monolithic one also attracts many interests in the last decade; see, e.g., [10,21,8,32,13,38,58,11,27,46,45,41]. Compare to the flexibility of the partitioned approach, where existing fluid and structure sub-problem solvers can be directly reused or adapted in an iterative manner, the monolithic one behaves more stable and robust by dealing with the coupled nonlinear FSI system in an all-at-once manner. Formally speaking, we apply Newton's method (see [24]) in an outer iteration dealing with nonlinearities originated from the domain movements, convection terms, material laws, transmission conditions and stabilization parameters (that may depend on the solution itself); as a price to pay, at each Newton iteration, a large linearized system is to be solved efficiently.

In the monolithic approach, the linearization of the nonlinear coupled system turns out to be a nontrivial task and requires tedious work on both the analytical derivation and computer implementation. One difficulty considered in this work results from the hyperelastic nonlinear material law as for the thick-walled artery with the media and adventitia layer (see [40,31]), for which the evaluation of the second Piola–Kirchhoff tensor and the material elasticity tensor demands heavy amount of computational effort in each Newton iteration; see, e.g., [39,15] for an introduction on

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the basic tools used to derive these quantities under the Lagrangian framework and e.g., [4] for the simulation of such arterial tissues. Thanks to our previous work in [52], the linearization for the hyperelastic models tackled in a partitioned FSI solver is reused in this work. Another difficulty stems from the fluid domain movement handled by the Arbitrary-Lagrangian–Eulerian (ALE) method, where the fluid domain displacement is introduced as an additional variable; see, e.g., [44,29,25,50]. To formalize the derivative of the fluid sub-problem with respect to the fluid domain displacement, the domain mapping (see, e.g., [72]) and shape derivative calculus (see, e.g., [13,2]) are two typical robust approaches mainly considered so far. In the domain mapping approach, the fluid sub-problem is mapped to the one on the reference (initial) fluid domain via the ALE mapping, that matches the Lagrangian structure domain on the interface for all the time. Therefore, the FSI transmission conditions are defined on the unchanged interface between the fluid and structure reference domains. By transforming the fluid sub-problem from the current domain (ALE framework) to the reference domain (Lagrangian framework), the fluid domain deformation gradient tensor and its determinant arise, which leads to a formulation similar to the one under the Lagrangian framework as usually adopted in continuum mechanics. Thus, for the fluid sub-problem, we follow the same approach to compute the directional derivative with respect to the fluid domain displacement (see related techniques in, e.g., [39,15]) as we used for the hyperelastic equations in [52]. In the second approach based on a shape derivative technique (see, e.g., [64]), the derivative of the fluid sub-problem is then evaluated by computing the directional derivative with respect to the change of geometry (a small perturbation) on the current domain; see also this technique employed by the partitioned Newton's method in [22,76]. We mention that we use the first approach in this work.

In addition to the effort on the linearization of the coupled nonlinear system, the monolithic solver requires properly designed preconditioners and solvers (as inner iteration) for the linearized coupled FSI system at each Newton iteration and may demand more effort. In [58], the preconditioned Krylov subspace method (see, e.g., [60]) and geometrical multigrid method (see, e.g., [37]) with a Vanka-like smoother are employed to solve the linearized and discretized 2D FSI system using the high order Q_2-P_1 stabilized finite element pair. For the complex 3D geometries and unstructured meshes, in [32], the GMRES method (see [61]) accelerated by the block Gauss–Seidel preconditioner is considered, for which the block inverse is approximated by smoothed aggregation multigrid (see, e.g., [62]) for each sub-problem. In order to improve the performance, a monolithic FSI algebraic multigrid (AMG) method using preconditioned Richardson iterations with potentially level-dependent damping parameters as smoothing steps is further developed therein. Besides, the monolithic solver is shown capable of utilizing parallel computing resources. In [21], parallel preconditioners of the coupled problem based on the algebraic additive Schwarz (see, e.g., [67]) preconditioners for the sub-problems are built for both the convective explicit and geometry–convective explicit time discretized FSI systems. As a 2D counterpart, in [10], a one-level additive Schwarz preconditioner (see, e.g., [67]) for the linearized system is considered for the fully implicit time discretized FSI system, that is based on a sub-domain preconditioner constructed on an extension of a non-overlapping sub-domain to its neighbors.

In this work, we focus on the development and comparison of different monolithic solution methods, namely, the Krylov subspace methods preconditioned by the block LU decomposition (see [60]) of the coupled system, the AMG and algebraic multilevel (AMLI [5,6,69,49], also referred to as K-cycle [56,57]) method, applied to the coupled FSI system with nearly incompressible hyperelastic models (see [40,31]). Our solution methods are mainly based on a class of special AMG methods developed in [47,70,71], for the discrete elliptic and saddle point problems, respectively, where the robust matrix-graph based coarsening strategies are proposed in a (pure) algebraic manner. This class of AMG methods have been applied to the sub-problems in the fluid–structure interaction simulation; see [76,75,52]. Particularly in our recent work [52], we have developed this approach by carefully choosing the effective smoothers: Braess–Sarazin smoother (see [16,77]) and Vanka smoother (see [68,74]), for the linearized Navier–Stokes equations under the ALE framework and hyperelastic equations under the Lagrangian framework, respectively. In order to further extend this class of AMG methods to the monolithic coupled FSI system after linearization, the two essential components in the AMG methods, the coarsening strategy and the smoother, for the coupled system are to be developed. Namely, the robust coarsening strategy using the stabilized Galerkin projection is constructed based on the inf–sup condition (see, e.g., [17,33]) on coarse levels for the indefinite sub-problems. By this means, we obtain the stabilized coupled systems on coarse levels. The effective smoother is designed by damped block Gauss–Seidel iterations applied to the coupled system, that are based on the AMG cycles for the mesh movement, fluid and structure sub-problem, respectively. According to our numerical experiments, we observe the robustness of the damping parameter with respect to the AMG levels and different hyperelastic models adopted in the FSI simulation. As a variant of our coupled AMG method, we further consider the AMLI method for the

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