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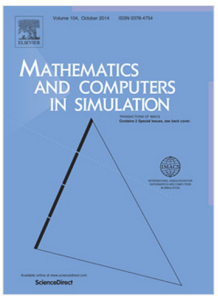

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M.P. Rajan, G.D. Reddy

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A regularized iterative scheme for solving singularly perturbed elliptic PDE

M. P. Rajan and G. D. Reddy
School of Mathematics
Indian Institute of Science Education and Research Thiruvananthapuram
CET Campus, Thiruvananthapuram-695 016, Kerala, India
Email: rajanmp@iisertvm.ac.in; reddygdamodar@gmail.com

Abstract

In this paper, we consider a class of singularly perturbed elliptical problems with homogeneous boundary conditions. We propose an iterative method for solving such problems. Convergence analysis and error estimate are derived. The proposed methodology is validated with the numerical results, and is also compared with well-known Shishkin scheme. The study demonstrates that the proposed regularized scheme has an edge over traditional numerical schemes.

AMS Classification: 65M60; 65M15; 65M12
Key words: Singular Perturbation; Elliptic PDE; Regularization; Iterative Method; Boundary Value Problem

1 Introduction

The study of singularly perturbed problems is an interesting problem to the researchers for many decades and many numerical methods were proposed in literature to solve such problems. A survey of such problems is given in M. Kadalbajoo, [5]. In this paper, we are concerned with solving a special class of singularly perturbed elliptic problems with homogeneous boundary conditions. Although various numerical schemes are available in literature (cf. [1, 5, 7, 9, 11, 15] and reference there in) to find the approximate solutions of such problems, the discretization of the PDE often leads to a highly ill-conditioned system, which results in unstable solutions. To circumvent this problem, we employ regularization techniques and try to achieve a more stable approximate solution to the given problem. We consider the following singularly perturbed PDE:

$$-c^2\frac{\partial^2 u}{\partial x^2} - d^2\frac{\partial^2 u}{\partial y^2} + a(x,y)u = f(x,y) \text{ in } \Omega, \tag{1.1}$$

$$u = 0 \text{ on } \partial\Omega, \tag{1.2}$$

where, $\partial\Omega$ is boundary of $\Omega = (0,\pi)\times(0,\pi)$, $c, d \in \{\epsilon, 1\}$, $c, d \neq 1$, $0 < \epsilon << 1$ is a parameter; $f(x,y)$ is a smooth function bounded on $\bar{\Omega} = \Omega \cup \partial\Omega$, and $0 < \gamma^2 \leq a$ is either a constant or a positive function, for some constant γ. The model that we have considered here is a prototype for many practical problems (cf. [5, 13]). In order to analyze the problem very closely, we write the equation (1.1) in compact form as:

$$Tu = f, \tag{1.3}$$

where, $T :\equiv -c^2\frac{\partial^2}{\partial x^2} - d^2\frac{\partial^2}{\partial y^2} + a$ is an operator acting between suitable Hilbert spaces. We assume that f is at least twice continuously differentiable in $\bar{\Omega}$ so that $L^2(\bar{\Omega})$ can be used as and when required. We further note that when the