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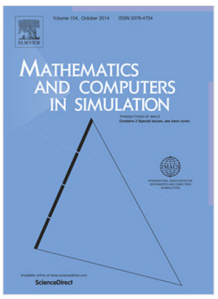

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Robert Eymard, Pierre Feron, Cindy Guichard

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Family of convergent numerical schemes for the incompressible Navier-Stokes equations

Robert Eymard[1], Pierre Feron[2], Cindy Guichard[3]

Abstract

This paper presents the common mathematical features which are leading to convergence properties for a family of numerical schemes applied to the discretisation of the steady and transient incompressible Navier-Stokes equations with homogeneous Dirichlet's boundary conditions. This family includes the Taylor-Hood scheme, the MAC scheme, the Crouzeix-Raviart scheme generalised into the Hybrid Mixed Mimetic scheme, which can be combined with a variety of discretisations for the nonlinear convection term, each of them being more efficient than the others in particular situations. We provide tools for analyzing all the combined methods, and proving their convergence to a weak solution of the problem.

Keywords: incompressible Navier-Stokes equations, Gradient Discretisation Method, convergence analysis

1. Introduction

For the approximation of the incompressible Navier-Stokes equation, most of the numerical schemes are nonlinear extensions of schemes applying to the linear incompressible Stokes equations. Three of them are of particular importance.

1. The Taylor-Hood scheme [20] is the prototype of conforming finite element methods on general simplicial grids. The convergence of this method (among more general conforming or nonconforming methods) with the skew symmetric approximation of the convection term (detailed in the Appendix) is proved for example in [21].
2. The Marker-And-Cell (MAC) scheme, introduced in [14], is one of the most popular methods in the engineering framework [19, 22] for the approximation of the Navier-Stokes equations on structured Cartesian grids. Its convergence properties, partly provided by the pioneering works [17, 18], are detailed in [11].
3. The Crouzeix-Raviart scheme [3] is a non conforming scheme on general simplicial grids, whose advantage is to provide fluxes and exact mass balances in the simplices. This scheme can be extended to general polyhedral meshes, using the Hybrid Mixed Mimetic (HMM) methods that include in particular the Mimetic Finite Difference schemes [1].

It is proved in [5] that all of them are gradient discretisation methods for which general properties of discrete spaces and operators are allowing common convergence and error estimates properties, in the case of the steady and transient incompressible Stokes equations. Turning to the Navier-Stokes problem,

[1]LAMA (UMR 8050), UPEM, UPEC, CNRS, F-77454, Marne-la-Vallée, France. Robert.Eymard@u-pem.fr
[2]LAMA (UMR 8050), UPEM, UPEC, CNRS, F-77454, Marne-la-Vallée, France. Pierre.Feron@u-pem.fr
[3]ANGE project-team, Laboratoire Jacques-Louis Lions (LJLL), UMR 7598 CNRS, Sorbonne Universités, UPMC Univ. Paris 6, and INRIA de Paris, and CEREMA, France. guichard@ljll.math.upmc.fr