Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 144 (2018) 235–246
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original Articles

Pricing of defaultable options with multiscale generalized Heston's stochastic volatility

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Received 18 October 2016; received in revised form 29 June 2017; accepted 23 August 2017
Available online 9 September 2017

Abstract

The possibility of default risk of an option writer becomes a more important issue in over-the-counter option market when systemic risk increases. It is desirable for the option price to reflect the default risk. On the other hand, it is known that a single scale, single factor stochastic volatility model such as the well-known Heston model would not price correctly in- and out-of-the money options. So, this paper studies the pricing of defaultable options under a multiscale generalized Heston's stochastic volatility model introduced by Fouque and Lorig (2011) to resolve these issues. We derive an explicit solution formula for the defaultable option price and investigate the characteristics of the resultant price in comparison to the price under the original Heston model.
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Keywords: Default risk; Option pricing; Stochastic volatility; Heston model; Multiscale

1. Introduction

Since the prices of many derivatives including vanilla and exotic options under the Black–Scholes model [1] can be derived in an analytic closed form, the model has been popularly used in both academia and industry. Despite of this advantage, however, there exist some drawbacks due to the assumption of constant volatility. For example, the model cannot capture the volatility smile/skew phenomenon which is often observed in financial market. To avoid this weakness, Heston [5] proposed an alternative model in which the constant volatility part of the Black–Scholes model is replaced by a diffusion process called the Cox–Ingersoll–Ross (CIR) process. Consequently, the Heston model can relax the weaknesses of the Black–Scholes model while it leads to an analytic pricing formula for various options, especially, European vanilla options.

When the off-exchange trading is carried out without any formal place nor any supervision, it is called over-the-counter (OTC) trading. While the transaction on exchange guarantees the required fulfillment of contract between

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http://dx.doi.org/10.1016/j.matcom.2017.08.005
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the two parties, the transaction in OTC market does not. For instance, the option writer may not keep the promise of payment at maturity. In other words, the option holder is vulnerable to the default risk caused by the counter party in OTC market. So, it is important to study the pricing of the 'defaultable' (vulnerable) options related to the default risk. A brief literature review about the defaultable options is as follows. Johnson and Stulz [6], and Klein [7] priced the European style defaultable option under the Black–Scholes model [1]. Lee et al. [9] studied the defaultable option following the framework of Klein [7] but under the Heston stochastic volatility model. In particular, they obtained a closed form pricing formula for the defaultable option under the Heston model. Yang et al. [12] obtained a pricing formula of the defaultable option under the fast mean-reverting stochastic volatility model of Fouque et al. [3].

Even if the Heston model is quite successful and frequently adopted in the study of financial modeling, however, there is a room to be improved because it is still a single factor volatility model as discussed by Zhang and Shu [13], Fouque and Lorig [2] and etc. In general, it is not sufficient to calibrate the suitable implied volatility surface through various expiration dates and strike prices under stochastic volatility models with a single diffusion in its volatility part. To overcome this weakness, Fouque and Lorig [2] proposed a modified version of the original Heston model by introducing another scale of volatility factor which is driven by a fast mean-reverting process. Based on this formulation, it is confirmed that the resultant model can give more flexible fitting performance related to the calibration of the implied volatility. So, this paper uses this multiscale framework to price European defaultable options exposed to the default risk. The contribution of this paper is that we obtain an explicit analytic formula of the option price under a multiscale generalized Heston model and we show how the model adjusts the defaultable option price of the original Heston model.

The rest of this paper is organized as follows. Section 2 describes a generalized multiscale Heston model. In Section 3, we review the known closed form formula for defaultable options under the Heston model and extend the formula under the extended Heston model. In Section 4, we compare numerically the original Heston price and the extended price to investigate the resultant correction effect to the Heston option price as well as the Heston implied volatility surface. Section 5 concludes.

2. Model formulation

In this paper, the price S_t of an underlying asset of an option is assumed to satisfy a multiscale generalized Heston's stochastic volatility model introduced by Fouque and Lorig [2], where the volatility is driven by an Ornstein–Uhlenbeck process with a fast mean-reverting factor. It is given by the stochastic differential equation (SDE)

$$\begin{aligned} dS_t &= rS_t dt + f(Y_t)\sqrt{Z_t}S_t dW_t^s, \\ dZ_t &= \kappa(\theta - Z_t)dt + \sigma_z\sqrt{Z_t}dW_t^z, \\ dY_t &= \frac{Z_t}{\epsilon}(m - Y_t)dt + u\sqrt{2}\sqrt{\frac{Z_t}{\epsilon}}dW_t^y, \end{aligned} \tag{1}$$

under the risk-neutral measure (Fouque and Lorig [2]), where r is risk-free interest rate, and κ and ϵ are parameters to determine the rate of mean-reversion of the processes Z_t and Y_t, respectively. Especially, ϵ is assumed to be positive and small so that the stochastic process Y_t is fast mean-reverting. σ_z and u are positive constants. Additionally, f is an unspecified bounded function.

On the other hand, the value V_t of the option writer's total asset is modeled by the SDE

$$dV_t = rV_t dt + \sigma_v\sqrt{Z_t}V_t dW_t^v, \tag{2}$$

where σ_v is a positive constant.

As the processes W_t^s, W_t^z, W_t^y and W_t^v are (dependent) Brownian motions, the instantaneous correlations between each two processes are denoted by

$$\begin{aligned} &d\langle W_t^s, W_t^z\rangle = \rho_{sz}dt, \quad d\langle W_t^s, W_t^y\rangle = \rho_{sy}dt, \quad d\langle W_t^s, W_t^v\rangle = \rho_{sv}dt, \\ &d\langle W_t^z, W_t^y\rangle = \rho_{zy}dt, \quad d\langle W_t^z, W_t^v\rangle = \rho_{zv}dt, \quad d\langle W_t^y, W_t^v\rangle = \rho_{yv}dt. \end{aligned}$$

Following the definition of Klein [7], the payoff function of a vulnerable call option is given by

$$h(S_T, V_T) = (S_T - K)^+\left(1_{\{V_T \ge D^*\}} + 1_{\{V_T < D^*\}}\frac{(1-\alpha)V_T}{D}\right) \tag{3}$$

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