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Multi-almost periodicity in semi-discretizations of a general class of neural networks

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Abstract

In this paper, we present multi-almost periodicity of a general class of discrete-time neural networks derived from a well-known semi-discretization technique, that is, coexistence and exponential stability of 2^N almost periodic sequence solutions of discrete-time neural networks subjected to external almost periodic stimuli. By using monotonicity and boundedness of activation functions, we construct 2^N close regions to attain the existence of almost periodic sequence solutions. Meanwhile, some new and simple criteria are derived for the networks to converge exponentially toward 2^N almost periodic sequence solutions. As special cases, our results can extend to discrete-time analogues of periodic or autonomous neural networks and hence complement or improve corresponding existing ones. Finally, computer numerical simulations are performed to illustrate multi-almost periodicity of semi-discretizations of neural networks.

Keywords: Neural networks; Semi-discretization; Multi-almost periodicity; Almost periodic sequence; Exponential stability

1. Introduction

In recent years, the dynamical behaviors of neural networks with or without delays have been widely investigated and correspondingly applied to associative memories and pattern classification, e.g., [2–8,11,13,17,18,25–27]. Several methods have been correspondingly proposed for designing associative memories in both continuous-time neural networks and discrete-time neural networks [2,5,23,24]. As we know, in associative memory neural network, the addressable memories or patterns are stored as stable equilibria or stable periodic orbits. Thus, in designing associative memories, it is an important task to discuss convergence analysis and coexistence of multiple equilibria or multiple periodic orbits. We can refer to [2,4,5,25,26] and so on.

For purposes of computer simulation and digital implementation, discrete-time analogues of neural networks have been derived from a semi-discretization technique with the value of the time-step fixed and it is shown that the analogues preserve the convergence dynamics of their continuous-time counterparts. These results have been attained

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in [15,16,19–21]. For nonautonomous neural systems (including almost periodic or periodic cases), the authors in [15,16] further to investigate discrete-time analogues of neural networks and attain the existence and global exponential stability of almost periodic-type sequence solutions.

However, to the best of the authors' knowledge, few papers deal with multi-almost periodicity of discrete-time neural networks. Hence, for a class of general neural networks [8,14] expressed as

$$\frac{du^i(t)}{dt} = -c_i(t)u^i(t) + \sum_{l=1}^{M}\sum_{j=1}^{N} a_{ijl}(t)g_j(u^j(t-\upsilon_{ijl})) + I_i(t),$$

we should consider corresponding discrete-time analogue as follows:

$$\begin{cases} u^i(n+1) = u^i(n)\exp(-c_i(n)) + \left(\dfrac{1-e^{-c_i(n)}}{c_i(n)}\right) \times \left\{\displaystyle\sum_{l=1}^{M}\sum_{j=1}^{N} a_{ijl}(n)g_j(u^j(n-\upsilon_{ijl})) + I_i(n)\right\}, \\ u^i(s) = \psi^i(s), \quad s = -\upsilon, -\upsilon+1, \cdots, 0. \end{cases} \tag{1}$$

where $i \in \mathcal{N} := \{1, 2, \cdots, N\}$, υ_{ijl} are assumed to be integers and $\upsilon := \max_{i,j,l}\{\upsilon_{ijl}\} > 0$. System (1) can be derived easily from the semi-discretization technique adopted by [15,16,19–21]. It is obvious that system (1) includes discrete-time neural networks considered by [15,20] as its special cases. Our main aim is to study multi-almost periodicity of above discrete-time neural networks, that is, coexistence of 2^N almost periodic sequence solutions and its local exponential stability. Our results are completely different from most of the existing results in [15,16,19–21]. Particularly, when system (1) degenerates into the periodic or autonomous system, our results extend the related results in [24] to stability analysis of multiple periodic orbits or multiple equilibrium points.

The rest of this paper is organized as follows. In Section 2, we shall make some preparations by giving some definitions, lemmas and basic facts of almost periodic sequences. Meanwhile, we establish the boundedness of discrete-time neural networks under almost periodic stimuli. In Section 3, by using the property of almost periodicity and inequality technique, we establish some new criteria for the existence and exponential stability of 2^N almost periodic sequence solutions. We also apply our results to periodic or autonomous neural networks and hence improve or complement existing ones correspondingly. Finally, computer numerical simulations are presented to illustrate our results.

2. Preliminary

Denote $\mathbb{Z}$ as the set of all integers and $\mathbb{Z}^+$ as the set of nonnegative integers, $[a, b]_{\mathbb{Z}} := \{a, a+1, \cdots, b\}$, where $a, b \in \mathbb{Z}$ and $a \le b$. Let $S([-\upsilon, 0]_{\mathbb{Z}}, \mathbb{R}^N)$ be the Banach space of all functions $\psi(s) = (\psi^1(s), \psi^2(s), \cdots, \psi^N(s))$ mapping $[-\upsilon, 0]_{\mathbb{Z}}$ into $\mathbb{R}^N$ with norm defined by $\|\psi\| = \max_{i\in\mathcal{N}}\{\sup_{s\in[-\upsilon,0]_{\mathbb{Z}}}|\psi^i(s)|\}$. For any given initial condition $\psi \in S([-\upsilon, 0]_{\mathbb{Z}}, \mathbb{R}^N)$, we denote by $\{u(n;\psi)\}$ the sequence solution of system (1). Let ℓ be a given nonnegative integer and $u_\ell \in S([-\upsilon, 0]_{\mathbb{Z}}, \mathbb{R}^N)$ be defined by $u_\ell(s) = u(\ell+s)$, $s \in [-\upsilon, 0]_{\mathbb{Z}}$. Then we can rewrite the initial condition as $u_0 = \psi \in S([-\upsilon, 0]_{\mathbb{Z}}, \mathbb{R}^N)$. For convenience, we present some important facts about almost periodic sequences. For corresponding resources, we can refer to [1,9,22].

Definition 1. [22] A real valued sequence $\{x(n)\}$, $n \in \mathbb{Z}$ is said to be almost periodic if for any $\epsilon > 0$ there exists a relatively dense set of ϵ-almost periods, i.e., there exists a natural number $L(\epsilon)$ such that for an arbitrary $m \in \mathbb{Z}$, one can find an integer $p \in [m, m+L(\epsilon)]$ for which the following inequality holds:

$$|x(n+p) - x(n)| < \epsilon.$$

Theorem 1. [22] *Assume that $\{x(n)\}$, $\{y(n)\}$, $n \in \mathbb{Z}$ are any two almost periodic sequences. Then the following affirmations are true:*

- *If $f(\cdot)$ is continuous on close interval containing the range of $\{x(n)\}$, then $\{f(x(n))\}$ is almost periodic. Any almost periodic sequence $\{x(n)\}$ is bounded;*