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Original Article

Spatial system estimators for panel models: A sensitivity and simulation study

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Abstract

Panel models are popular models in applied sciences and the question of spatial errors has recently created the demand for spatial system estimation of panel models. In this paper we propose new diagnostic methods to explore if and how the spatial components will make significant differences of spatial estimates from non-spatial estimates of seemingly unrelated regression (SUR) systems. We apply a local sensitivity approach to study the behavior of spatial ordinary or generalized least-squares estimators in two spatial SUR system models: a spatial autoregressive regression model with SUR errors and a SUR model with spatial errors. Using matrix differential calculus we establish a sensitivity matrix for the spatial panel models. We show how a first-order Taylor approximation based on the non-spatial ordinary or generalized least-squares estimators can be used to approximate the least-squares estimators in spatial SUR models. In a simulation study we examine the approximation results and demonstrate their quality. We also try to find whether the SUR variance or the neighbourhood weight matrix has more impact on the estimates and their approximations.

Keywords: Generalized and ordinary least-squares estimators; Panel systems with spatial components; Seemingly unrelated regression models; Spatial autoregressive regression and spatial error models; Taylor approximations

1. Introduction

A number of spatial models and applications in statistics and econometrics have been studied for over the past three decades. Different estimation methods are used, including maximum likelihood estimation, Bayesian estimation, spatial two-stage least-squares, generalized method of moments and matrix exponential methods; see e.g. Paelinck and Klaassen (1979), Anselin (1988, 2010), Kelejian and Prucha (1997, 1998, 2007), Das et al. (2003), Florax and Van der Vlist (2003), Haining (2003), Lee (2003, 2004), Bivand et al. (2008), LeSage and Polasek (2008), LeSage and Pace (2009), Liu et al. (2012).

The usual cross-sectional spatial autoregressive (SAR) model is given by

$$y = \rho W_n y + X\beta + u, \quad u \sim N[0, \sigma^2 I_n], \tag{1}$$

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where y is an $n \times 1$ vector of cross-sectional observations, ρ is a scalar correlation parameter for the observations, W_n is an $n \times k$ neighbourhood weight matrix, X is an $n \times k$ explanatory matrix, β is a $k \times 1$ vector of parameters, and u is an $n \times 1$ vector of errors, which follow a normal distribution with mean 0 and variance matrix $\sigma^2 I_n$, and I_n is an $n \times n$ identity matrix.

In a similar way the spatial error (SEM) model is given by

$$y = X\beta + e, \quad e = \theta W_n e + u, \quad u_t \sim N[0, \sigma_u^2 I_n], \tag{2}$$

where θ is a scalar parameter for the spatial errors.

It is very important and indeed useful to examine the sensitivity of these estimators in terms of a minor change in the spatial correlation parameters. Sensitivity analysis is made for a number of issues and applications in econometrics and statistics; see e.g. Polasek (1984) for results in the Bayesian framework, Magnus and Vasnev (2007) for a systematic study with uses in testing and especially the local sensitivity matrix, Liu and Neudecker (2008) for a local sensitivity of the restricted least-squares estimator in the linear model, Liu et al. (2012) for an attempt to deal with the cross-sectional spatial models, and Paula et al. (2012) for related approaches.

In the present paper, we consider generalized least-squares (GLS) methods, due to their "simplicity" in the sense that an analytical expression can be made available for the estimators. We essentially use the local sensitivity matrix as advocated by Magnus and Vasnev (2007), Liu et al. (2012), which only requires the first-order derivatives and underpins the Taylor approximations.

Sensitivity analysis with respect to ρ in a SAR model means that we are interested in the behavior of the estimators of β upon a small change of ρ. The numerical computation of the "spatial filter" estimator of the SAR model uses the spatial filter matrix $R = I - \rho W$, which acts as a filter in the reduced form of the SAR model. Spatial estimators are a function of the spatial correlation parameter ρ and the spatial neighbourhood matrix W, which tends to become a large-scale matrix in large spatial panel systems.

Our question is if "good" approximations of simple spatial estimators exist to justify a reasonable sensitivity analysis (or make a spatial diagnostics or approximation without employing a time consuming spatial estimation procedure), and if so, what estimators and what estimation approaches should be considered for use? A previous study of the sensitivity of spatial estimators in the cross-sectional SAR and SEM models is made in Liu et al. (2012). It is shown that good approximations exist for small values of ρ. Here we may assume ρ is known (or estimated already).

Note that, in the meantime, panel models have become increasingly important and different estimators in such models with spatial components have also been studied; see e.g. Elhorst (2003, 2010), Kapoor et al. (2007), Anselin et al. (2008), Baltagi (2008), Kakamu et al. (2008, 2012), Kakamu and Wago (2008), Lee and Yu (2010).

So, we build upon Liu et al. (2012) and extend a system of panel models to a seemingly unrelated regression (SUR) system with spatial errors in two ways: One is a SAR model with SUR errors (SAR-SUR) and the other is a SUR model with spatial errors (SUR-SEM). First, we propose a system GLS estimator with spatially filtered variables, which is the SF-GLS estimator. We show that it can be expanded in a first-order Taylor expansion around the non-spatial GLS estimator of a non-spatial regression model. The second system estimator is the reduced form RF-OLS estimator, which is actually an ordinary least-squares (OLS) estimator and amounts to spatially transform all dependent and independent variables.

While in a cross-sectional SAR or SEM model we have to explore the sensitivity with respect to only one spatial parameter, in the system case we need a vector of spatial parameters, with each spatial correlation parameter for a cross-sectional sample. To get the sensitivity result based on matrix derivatives for a vector of correlation parameters, we use a simple trick that is found in e.g. Neudecker et al. (1995a,b). As we are only interested in the derivative with respect to the diagonal matrix, we first derive the matrix-to-matrix derivative and then in the last step we employ the general result that a diagonal derivative can be obtained by post-multiplying the matrix-to-matrix derivative with the selection matrix J, which is presented in Appendix A.

In addition, we investigate if simplifications of the system sensitivity results are possible when we make the simplifying assumptions that there only exists one common spatial correlation parameter. In the SAR-SUR system case we gain no new insights by doing these simplifications, but luckily, we find for the SAR-SEM model nice interpretations in the line of global sensitivity analysis as in Leamer (1978). The spatial correlation parameter traces out a hyper-curve between two simpler non-spatial estimators. For the simulation study we develop a basic design involving the number of observations, the neighbourhood matrix W and the SUR covariance matrix Σ. We focus on the estimators of β and their sensitivity behavior upon a small change of ρ. We discuss methods used to measure