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Upgrading the 1-center problem with edge length variables on a tree

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ABSTRACT

This paper discusses upgrading the 1-center problem on networks, which tries to change the lengths of the edges within certain bounds and find the best place for 1-center with respect to the new lengths so that the objective value is minimized. As this problem is $\mathcal{NP}$-hard on general graphs, the problem is considered where the underlying graph is a tree. It is mentioned that this problem is solvable in polynomial time by solving a series of linear programs. A combinatorial algorithm with $O\left(n^2\log(n)\right)$ time complexity is proposed for the equal cost case, where n is the number of vertices of the tree. It is also shown that the problem is solvable in $O\left(n^2\log(n)^2\right)$ time on an unweighted tree, i.e., all vertex weights are equal to one, but the costs are arbitrary.

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1. Introduction and related problems

Unlike to classical location problems which are concerned with finding optimal locations for facilities, the upgrading problems deal with changing the parameters of a given problem within certain limits and a budget constraint so that the optimal objective value with respect to the modified parameters is minimized. In this paper, upgrading approach is applied to the 1-center problem on networks with edge lengths variables.

The classical 1-center problem aims to find the best place on the given network so that the maximum weighted distance from this place to vertices is minimized. Mathematically speaking, given a graph $G(V,E)$ with vertex weights $w_v \in R^+$, $v \in V$ and edge lengths $\ell_e \in R^+$, $e \in E$, the objective function of the classical 1-center problem can be written as below

$$\min_{v\in V}\max_{u\in V} w_u d_{\boldsymbol{\ell}}(v,u), \tag{1}$$

or

$$\min_{v\in V} z(\boldsymbol{\ell},v), \tag{2}$$

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where $z(\boldsymbol{\ell}, v) = \max_{u \in V} w_u d_{\boldsymbol{\ell}}(v, u)$, $\boldsymbol{\ell}$ is the vector of edge lengths ℓ_e, $e \in E$, and $d_{\boldsymbol{\ell}}(v, u)$ denotes the shortest path distance between u and v with respect to the edge lengths vector $\boldsymbol{\ell}$. This problem has been studied in details by some authors specially over the past two decades.

However, upgrading the 1-center problem is a variant of the 1-center problem on networks where a given budget is assigned in order to decrease the length of edges within certain limits. In upgrading 1-center problem we are allowed to change the network and then find the best place for the center. In other words, upgrading the 1-center problem is aimed at finding new values for the edge lengths and a new center associated with these new parameters so that this center is the best over all allowed parameters.

Some authors have already applied upgrading approach to several classical optimization problems. Fulkerson and Harding [1] and Hambrusch and Hung-Yi Tu [2] have independently dealt with upgrading the shortest and longest path problems. Phillips [3] has investigated upgrading the network flow problem. Up and downgrading the 1-median problem has been investigated in [4] and [5] by Gassner. She has also developed upgrading the 1-center problem with vertex weights variables [6]. Sepasian and Rahbarnia [7] have solved upgrading 1-median on paths with a linear time algorithm. Some authors have also considered upgrading minimum spanning tree problem. Drangmeister et al. [8], Frederickson and Solis-Oba [9] and Krumke et al. [10] have developed upgrading Steiner tree and minimum spanning tree problems. Recently, Sepasian and Monabbati [11] have investigated upgrading the min–max spanning tree problem and developed some combinatorial algorithm to solve the problem.

In this paper we study upgrading the 1-center problem. Suppose we decide to choose the best location as 1-center according to the information (parameters) of a given area. However, we know that the efficiency of selected locations, everywhere established, can be improved with a fee for changing the parameters. Therefore, some locations may become the center of the network after changing the parameters. The location with the higher efficiency after the changes should be selected. So upgrading the 1-center problem is applicable as well as the classic 1-center problem provided the parameters are changeable. For example, before establishing a distribution center of goods or services, a fire station or a hospital, we can study the ability to reduce traffic and then start to set up.

Although some improvement problems investigated variable vertex weights, it seems the variable edge lengths to be more realistic. Because changing the vertex weights is hard as they represent the demand or client ranking (which are not in our control), while changing the traveling times, traffic connections, widen roads, etc., which are modeled as the edge lengths, is much easier [12].

An instance of upgrading the 1-center problem is given by a graph $G = (V, E)$ with vertex weights $w_v \in R^+$, $v \in V$ and edge lengths $\ell_e \in R^+$, $e \in E$. In upgrading the 1-center problem we decide to shorten the edges. A budget $B > 0$ is given in order to spend reducing the length of edges. Reducing each length ℓ_e is limited to a lower bound $\underline{\ell}_e > 0$, and c_e is the cost of reducing the length of e by one unit. Define the set $\mathcal{F} = \{\hat{\boldsymbol{\ell}} | \sum c_e \hat{\ell}_e \le B, \underline{\ell}_e \le \hat{\ell}_e \le \ell_e, e \in E\}$. Therefore, upgrading the 1-center problem can be formulated as

$$\min_{\hat{\boldsymbol{\ell}} \in \mathcal{F}} \min_{v \in V} z(\hat{\boldsymbol{\ell}}, v) = \min_{\hat{\boldsymbol{\ell}} \in \mathcal{F}} \min_{v \in V} \max_{u \in V} w_u d_{\hat{\boldsymbol{\ell}}}(v, u) \tag{3}$$

where $d_{\hat{\boldsymbol{\ell}}}(v, u)$ denotes the shortest path distance with respect to the new lengths $\hat{\ell}_e$, $e \in E$.

We introduce some basic notations and concepts which will be used throughout the paper. Let $T(V, E)$ be a tree and v be a vertex. $\mathcal{T}(v)$ denotes the collection of subtrees induced by deleting all edges joined to v. For a neighbor of v like $u \in V$, T^u denotes the subtree in $\mathcal{T}(v)$ containing u. We already show the distance between v and u by $d_{\boldsymbol{\ell}}(v, u)$ and $d_{\hat{\boldsymbol{\ell}}}(v, u)$ with respect to the initial vector edge lengths $\boldsymbol{\ell}$ and new vector edge lengths $\hat{\boldsymbol{\ell}}$, respectively. Similarly, $d_{\underline{\boldsymbol{\ell}}}(v, u)$ shows the distances with respect to the vector of lower bounds of edge lengths, $\underline{\boldsymbol{\ell}}$. $P[v, u]$ shows the path between the vertices v and u. Finally, n represents the number of vertices of the given network.

As we will see later, upgrading the 1-center problem is closely related to reverse 1-center problem. Let us briefly review this kind of problem.

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