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The skiving stock problem and its relation to hypergraph matchings

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ABSTRACT

We consider the one-dimensional skiving stock problem which is strongly related to the dual bin packing problem: find the maximum number of products with minimum length L that can be constructed by connecting a given supply of $m \in \mathbb{N}$ smaller item lengths $l_1, \ldots, l_m$ with availabilities $b_1, \ldots, b_m$. For this $\mathcal{NP}$-hard optimization problem, we investigate the quality of the proper relaxation by considering the proper gap, i.e., the difference between the optimal objective values of the proper relaxation and the skiving stock problem itself. In this regard, we introduce a theory to obtain the proper gap on the basis of hypergraph matchings. As a main contribution, we characterize those hypergraphs that belong to an instance of the skiving stock problem, and consider the special case of 2-uniform hypergraphs in more detail. Moreover, this particular class is shown to correspond one-to-one and onto a certain power set. Based on this result, the number of related non-isomorphic hypergraphs and their possible proper gaps can be calculated explicitly.

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1. Introduction and preliminaries

In this paper, we consider the one-dimensional skiving stock problem (SSP) [1,2] which is strongly related to the dual bin packing problem (DBPP) in literature (see e.g. [3–5]). In the classical formulation, $m \in \mathbb{N} := \{1, 2, \ldots\}$ different item lengths $l_1, \ldots, l_m$ with availabilities $b_1, \ldots, b_m$ are given, the so-called *item supply*. We aim at maximizing the number of products with minimum length L that can be constructed by connecting the items on hand, see Fig. 1.

Although representing similar problem statements, both denotations (DBPP and SSP) are separated in the following sense in literature[1] : the term DBPP rather refers to highly heterogeneous input lengths, meaning that the quantities b_i are very small (mostly even equal to one) for all $i \in I := \{1, \ldots, m\}$, whereas larger values of b_i are considered whenever the term SSP is used, in general (see the typology presented

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[1] This is the same differentiation as in the cutting context, where the *bin packing problem* and the *cutting stock problem* are considered.

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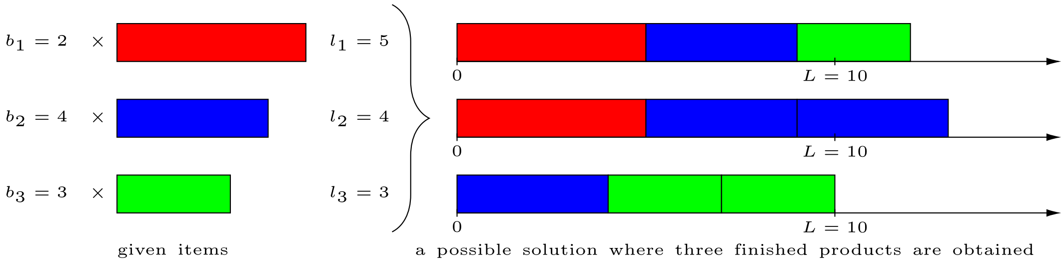

Fig. 1. A schematic for a skiving stock problem of small problem size.

by [6]). In other words, the DBPP focuses more on the individual character of each single item, and the SSP treats identical items (that are grouped together) collectively. Depending on the specific mathematical context both descriptions can be advantageous (as explained in the following sections).

The SSP can be considered as a natural counterpart of the extensively studied one-dimensional cutting stock problem (CSP) [7–11], where larger items have to be cut into smaller ones such that a given demand is satisfied. Although both problems share a certain common structure (e.g. as regards their input data), they are not dual formulations in the sense of mathematical optimization. Hence, the SSP represents an independent challenge in the field of discrete optimization.

For the case $b_i = 1$ $(i \in I)$, the considered optimization problem was firstly mentioned by Assmann et al. [3,12] and denoted by *dual bin packing problem*. Therein, the authors mainly investigate heuristic approaches and provide results regarding the quality and the average case behavior of the presented methods. Further contributions, especially in terms of exact approaches to the DBPP, have been studied in [4] and [5] where two branching algorithms are introduced. Based on practical preliminary thoughts [1], a generalization for larger availabilities $b_i \in \mathbb{N}$ $(i \in I)$ has been considered in [2] also introducing the term *skiving stock problem*. In that article, the author formulates a pattern-oriented model of the SSP with an infinite number of variables and provides first (numerical) results regarding the additive integrality gap of this optimization problem, i.e., the difference between the optimal values of the continuous relaxation of the SSP and the SSP itself. Further modeling approaches and detailed computational experiments have recently been presented in [13].

Despite being a relatively young field of research with only a limited number of publications, such computations are of high interest in many real world applications, e.g. industrial production processes [1,2], politico-economic problems [3,4] or wireless communications [14]. In particular, the SSP plays an important role whenever an efficient and sustainable use of given resources is desired. Referring to this, some main areas of applications are given by:

- recycling offcuts in industrial scenarios [2], even within a holistic framework as a combined cutting-and-skiving procedure [1,15],
- stimulating economic activity (e.g. in periods of recession) [4],
- efficient allocation of wireless users in a given frequency range by means of spectrum-aggregation based resource allocation [16,17].

Furthermore, also neighboring tasks, such as dual vector packing problems [18,19] or the maximum cardinality bin packing problem [5,20], are often associated with the DBPP. These formulations are of practical use as well since they are applied in multiprocessor scheduling problems [21] or surgical case plannings [22].

Usually, the abbreviation $E := (m, l, L, b)$ with $l = (l_1, \ldots, l_m)^\top$ and $b = (b_1, \ldots, b_m)^\top$ is used for an instance of the skiving stock problem. However, note that, whenever the individual character of each single

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