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Complexity of min–max–min robustness for combinatorial optimization under discrete uncertainty[☆]

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ABSTRACT

We consider combinatorial optimization problems with uncertain linear objective functions. In the min–max–min robust optimization approach, a fixed number k of feasible solutions is computed such that the respective best of them is optimal in the worst case. The idea is to calculate the set of candidate solutions in a potentially expensive preprocessing phase and then select the best solution out of this set in real-time, once the actual scenario is known. In this paper, we investigate the complexity of the resulting min–max–min problem in the case of discrete uncertainty, as well as its connection to the classical min–max robust counterpart, for many classical combinatorial optimization problems. Essentially, it turns out that the min–max–min problem is not harder to solve than the min–max problem, while producing much better solutions in general for larger k. In particular, this approach may mitigate the so-called price of robustness, making it an attractive alternative to the classical robust optimization approach in many practical applications.

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1. Introduction

Uncertain data can occur in many real-world optimization problems. Typical examples of parameters which can be uncertain are the costs and demands in the vehicle routing problem, the returns of assets in financial applications, arrival times of jobs in scheduling problems and many more. In this paper we study combinatorial optimization problems of the form

$$\min_{x\in X} c^\top x \tag{M}$$

where $X \subseteq \{0,1\}^n$ is the set of all feasible solutions and only the cost vector c is uncertain. Among others, we will consider the shortest-path problem, the spanning-tree problem, and the assignment problem with uncertain costs.

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One approach to handle these uncertainties is robust optimization, which was first introduced by Soyster [2] in 1973. The idea is to define an *uncertainty set* U which contains all scenarios of the uncertain parameters that are likely enough to be taken into account. The aim is to find a solution which is feasible for all scenarios in U and which optimizes the worst-case value over all scenarios in U. The robust optimization approach received increasing attention in the late 1990s. Kouvelis and Yu [3] studied various uncertain combinatorial optimization problems with finite sets of scenarios. Almost at the same time, Ben-Tal and Nemirovski analyzed the approach for convex problems where the uncertainty set is a cone or an ellipsoid [4,5]. Furthermore, El Ghaoui et al. studied semi-definite problems and least-square problems with uncertain data [6,7]. Some years later, Bertsimas and Sim introduced budgeted uncertainty sets and described and analyzed what they call the *Price of Robustness* [8].

In the case of cost uncertainty, the robust optimization idea leads to the well known min–max problem

$$\min_{x\in X} \max_{c\in U} c^\top x. \tag{M^2}$$

The latter problem is known to be too conservative for many practical applications, since considering all possible scenarios can lead to solutions which are far from optimal in many scenarios [8]. Moreover, Problem (M^2) is NP-hard for many classes of uncertainty sets, even for most combinatorial optimization problems that are tractable in their deterministic version. An overview over recent complexity results for the case of discrete uncertainty, i.e. for finite uncertainty sets, is given in [9].

To tackle these problems, many new approaches have been developed in the robust optimization literature recently. One of these approaches is the so called *adjustable robustness* that was introduced by Ben-Tal et al. [10]. The authors propose a two-stage model where the set of variables is decomposed into *here and now* variables x and *wait and see* variables y. The objective is to find a solution x such that for all possible scenarios there exists a y such that (x, y) is feasible and minimizes the worst case. This problem is known to be hard theoretically and practically. Therefore Bertsimas and Caramanis [11] introduced the concept of k-adaptability to approximate the latter problem. The idea is to compute k second-stage policies here-and-now; the best of these policies is chosen once the scenario is revealed. This idea was later used by Hanasusanto et al. [12] to approximate two-stage robust binary programs. The authors showed that if the uncertainty only effects the objective function, it is sufficient to calculate $n+1$ solutions to reach the exact optimal value of the two-stage problem. To the best of our knowledge the latter approach has not been studied for discrete uncertainty sets yet.

For the special case where no first stage exists, this idea was investigated in [1,13] in order to address combinatorial problems with uncertainty in the objective function. The resulting optimization problem is

$$\min_{x^{(1)},\dots,x^{(k)}\in X} \max_{c\in U} \min_{i=1,\dots,k} c^\top x^{(i)} . \tag{M^3}$$

The authors prove that (M^3) can be polynomially reduced to the deterministic problem (M) if $k \geq n+1$ and if U is a convex set over which a linear function can be optimized efficiently. Unfortunately, this result cannot be extended to the case of discrete uncertainty: different from the standard robust optimization approach (M^2), the uncertainty set U cannot be replaced by its convex hull in (M^3) without changing the problem.

In this paper, we concentrate on discrete uncertainty sets $U = \{c_1, \dots, c_m\}$. It was shown in [3,9] that (M^2) with discrete uncertainty is NP-hard for many combinatorial problems, even if the number m of scenarios is fixed, and that it is even strongly NP-hard if the number of scenarios is part of the input. On the other hand, for most of these problems pseudo-polynomial algorithms were found [3,14] for the case of fixed m. By reducing the min–max problem (M^2) to multicriteria optimization, it was further shown that many such problems admit an FPTAS [15].

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