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On the complexity of energy storage problems

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ABSTRACT

We analyze the computational complexity of the problem of optimally managing a storage device connected to a source of renewable energy, the power grid, and a household (or some other form of energy demand) in the presence of uncertainty. We provide a mathematical formulation for the problem as a Markov decision process following other models appearing in the literature, and study the complexity of determining a policy to achieve the maximum profit that can be attained over a finite time horizon, or simply the value of such profit. We show that if the problem is deterministic, i.e. there is no uncertainty on prices, energy production, or demand, the problem can be solved in strongly polynomial time. This is also the case in the stochastic setting if energy can be sold and bought for the same price on the spot market. If the sale and buying price are allowed to be different, the stochastic version of the problem is #P-hard, even if we are only interested in determining whether there exists a policy that achieves positive profit. Furthermore, no constant-factor approximation algorithm is possible in general unless P = NP. However, we provide a Fully Polynomial-Time Approximation Scheme (FPTAS) for the variant of the problem in which energy can only be bought from the grid, which is #P-hard.

1. Introduction

A problem that has been receiving increasing attention in recent years (see e.g. [1–4]) is that of optimally managing sources of renewable energy connected to the power grid, batteries, and potentially a household or some other form of energy sink. The presence of a battery is extremely valuable for peak shaving, time shifting, reduction of electricity price arbitrage, and to provide operating reserve, see [4–6]. The *energy storage* problem is that of deciding when to store, release, buy, and sell energy in this context. In this paper we focus on a basic subproblem of the complex decision problem introduced above. We call the basic subproblem *single-node* because it corresponds to the decision problem faced by a single energy-producing node in a smart grid, and does not explicitly take into account the goals of the system operator. The problem can

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naturally be modeled as a stochastic dynamic program. Existing works in the literature focus on the study of practically viable solution methods, typically through approximate dynamic programming approaches, and on the analysis of optimal or heuristic policies. To the best of our knowledge, no analysis of the complexity of this problem has been carried out. This work aims to fill this gap, identifying which characteristics of the problem make it difficult to solve from a theoretical point of view, i.e. no polynomial-time algorithm exists unless P = NP.

In this paper we give a general model for the single-node energy storage problem as a Markov decision process, and study its complexity. Our formulation is consistent with models in the recent literature. We show that the deterministic version of the problem can be cast as a minimum-cost flow problem and solved in strongly polynomial time, but the problem becomes #P-hard as soon as uncertainty is introduced, even for a restricted version of the problem in which energy can only be bought from the grid, i.e. selling on the spot market is not allowed. This hardness result holds for both determining the optimal policy and just determining the optimal policy value. In the general case, we show that even deciding whether the problem admits any policy achieving positive profit is #P-hard. The freedom of distributing energy from the renewable source or the battery to different devices during the same time period may reduce the difficulty of the problem, in the sense that if this possibility is removed or if it comes at a cost, the problem is weakly NP-hard already in the deterministic setting. Restricting the sale price on the spot market to be equal to the buying price makes the problem easier, because we provide a polynomial-time dynamic programming scheme for the stochastic case. This restriction on the price is well-studied in the literature (e.g. [4]), and it implies that the market is arbitrage-free. Our analysis suggests that the difficulty of numerically solving this restricted version of the problem originates from the complexity of handling the random processes, which can be very complicated in practice, rather than from the structure of the decision problem itself. The situation is considerably different than in the general case: the unrestricted version of the problem is intrinsically hard, i.e. it is #P-hard even when the random processes are very simple (independent random variables with support of size two). Finally, we show that no deterministic constant-factor approximation algorithm is possible in general, but we provide a Fully Polynomial-Time Approximation Scheme (FPTAS) for the case where selling to the grid is not allowed, indicating that solving the problem in a context with no sales may be easier than in the general case. An FPTAS is a deterministic ϵ-approximation algorithm that runs in polynomial time in the binary input size and $1/\epsilon$ for any $\epsilon > 0$, and it is considered the strongest possible result in terms of approximation.

Our model considers a single storage device, and models with multiple storage devices such as in [7] are at least as hard. This paper assumes that the states and transition probabilities of the Markov decision process are fully known: from a theoretical point of view this is a necessary assumption for exact computations (if we do not have access to such data, the problem is intractable in general), but it limits the practical relevance of the proposed algorithms. We show that if the probabilities are given implicitly by means of an oracle, instead of explicitly, even a version of the problem that is usually polynomial-time solvable may require an exponential number of oracle calls in the worst case, indicating that attempts at finding efficient algorithms are likely to fail unless specific structure is imposed on the stochastic processes.

From a methodological point of view, our FPTAS is a problem-specific extension of the framework of K-approximation sets and functions introduced in [8] to continuous (as opposed to discrete) state and action spaces, exploiting the piecewise linear structure of the objective function. This extension can easily be adapted to other continuous stochastic dynamic programs that share the same structure, i.e. piecewise linear convex costs (for a minimization problem) and affine transition function, yielding an FPTAS for those problems as well. The input data of the problem can be rational.

This paper is organized as follows. Section 2 is dedicated to a literature review. Section 3 introduces the mathematical model for the energy storage problem. Section 4 discusses our notation and a structural property of optimal policies for the problem, giving simplified models for two restrictions of the problem