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# Experimental and numerical study on an ultrasonic horn with shape designed with an optimization algorithm

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ABSTRACT

The paper presents the design and characterization of an axisymmetrical ultrasonic horn held by its circumference, with specified working frequency, amplification factor and nodal point position. To this purpose theoretical, experimental and computational methods are used in an efficient manner. The longitudinal waveform that ensures the imposed conditions is obtained in 2 stages: first it is determined theoretically according to conditions, then the working setup that gives the required waveform at resonance is designed. The waveform and corresponding horn shape are designed by using an optimization method based on Webster's horn equation, Hooke's law and variational calculus theory. The influence of added couplings at input and output in order to emulate transducer and tool is also treated. The separate optimization of these couplings allows setting different working frequencies for the ultrasonic horn system. Two 1-D methods, transfer-matrix and coupled-oscillators method and a 3-D finite element method are used in computer simulations. The proposed model of ultrasonic horn has the advantage of a more convenient placement of the nodal point, which is very important in industrial applications.


## 1. Introduction

Ultrasonic horns are mechanical devices used in order to amplify the ultrasonic signal by increasing the wave density of energy. Such amplification of the signal is fundamental in a great variety of ultrasonic applications in many technological processes of manufacturing industries, such as: automotive industry, food processing, medical equipment, material joining, etc. Among the most important technological applications, one can mention: welding, machining, cutting, drilling, cleaning, plastic welding, etc. [1–4].

Any ultrasonic equipment used in manufacturing activities often consists of four main parts: ultrasonic generator, transducer, ultrasonic solid horn (USH) (also referred to as 'sonotrode') and the tool. USH is a device that carries, to a high degree of efficiency, the acoustic energy from a transducer to the tool attached at the small end (Fig. 1).

The operation of such a system consists of an initial signal emitted by the transducer, with the amplitude $u_0$, that is amplified by the horn with a magnitude $q$. The amplification is ensured by the USH varying cross-section, from an initial value $S_0$, in the origin of the reference system, to a minimum value $S_L$, at the end of the horn The gain of USH is defined as the ratio of output amplitude $u_{out}$ to input amplitude $u_0$:

$$q = \left|\frac{u_{out}}{u_0}\right| \quad (1)$$

There are a great number of studies on USH, all of which consider horns of axially-symmetric shapes. At the same time, it is assumed that stress waves remain planar and the stresses are evenly distributed over the horn's cross-section. The cross-section variation in shape is given by different mathematical functions, such as: exponential, linear, conical, catenary, tapered, stepped, etc., which will be used according to the specific technological process [5–8]. The optimal performance of the ultrasonic manufacturing requires to give due consideration to the most important parameters that define the dynamics of the whole ultrasonic system [2,9–12].

With an ultrasonic system based on magnetostriction, the transducer and USH form an embedded mechanical system, and the operating frequency depends on the dimensions and elastic properties of both components [12,13]. The performance of the device can be improved by optimization studies on the USH. These studies are performed starting from different mathematical methods [14,15]. The transfer-matrix method is used to analyze the ultrasonic flexural modes of solid horns with exponential,

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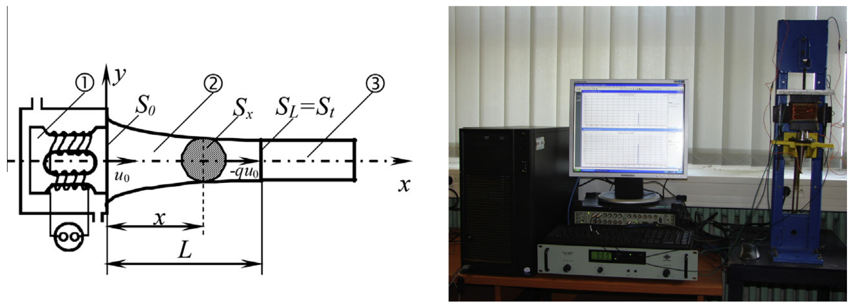


(a) (b)

**Fig. 1.** Ultrasonic system: (a) principle of USH working 1 – electromechanical transducer; 2 – ultrasonic horn; 3 – tool and (b) test set-up.

conical, and catenary profiles [7]. In other papers, the finite element method is used to study the microstructural processes at the interface between different tool parts [16] or at the cutting tool–chip interface [17].

Designing a circumference-held ultrasonic horn presents some specific issues as compared to longitudinally held horn. Separating the support point from the input surfaces of the acoustic horn allows separate and precise optimization of the behavior of the support. Input and output end coupling elements can be provided in order to obtain different working frequencies. The couplings can be designed separately from the body, as presented in the article.

The paper proposes an axisymmetrical ultrasonic horn that is held by its circumference at a given distance along its length. This requires that the node of the waves at the working frequency is fixed at the supporting position in order to cancel the influence of the support element on the longitudinal propagation of waves.

In order to optimize energy transfer, the working frequency is set as the resonance frequency for the working setup comprising the transducer, ultrasonic horn and tool. At this frequency, the gain of the USH is also set to a specific value. Thus, the working frequency and corresponding gain $q$ represent parameters with fixed values in the design of the USH.

The horn's mass is minimized with the above constraints, which yields an optimization problem. The optimization is carried starting from Webster's equation for longitudinal wave propagation along the horn's length. Resonance frequencies are obtained for different setups of the USH.

A series of 1D and 3D numerical simulations are used to validate the results obtained and further complete the horn design. In particular, since the design and optimization have been theoretically carried in the longitudinal wave propagation setup, a further finite element method analysis was necessary in order to check the validity of the model and give a full realistic 3D description of the behavior of the acoustic horn system, to ensure the longitudinal wave propagation condition was satisfied.

## 2. Theoretical considerations

Let us consider an USH with a variable cross-section. The propagation of acoustic longitudinal waves in the horn is described by Webster's horn equation [18,19]:

$$\frac{\partial^2 u(x,t)}{\partial x^2} + \frac{\partial u(x,t)}{\partial x}\frac{\partial}{\partial x}(\ln S_x) = \frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2}, \tag{2}$$

where $u(x,t)$ is the displacement of ultrasonic signal, $S_x$ is the cross-section area at distance $x$, and $c$ is the sound velocity in the horn's material.

The techniques employed to find the explicit analytical solutions of the linear partial differential equation are based on the method of separation of variables [20]:

$$u(x,t) = u(x)u(t) = u_x u_t. \tag{3}$$

By substituting solution (3) into Webster's equation (2), the following equation is obtained:

$$\frac{u''_x}{u_x} + \frac{d}{dx}(\ln S_x)\frac{u'_x}{u_x} = \frac{1}{c^2}\frac{\ddot{u}_t}{u_t}, \tag{4}$$

where $u''_x = d^2u_x/dx^2$, $u'_x = du_x/dx$, $\ddot{u}_t = d^2u_t/dt^2$.

The congruency of the left- and right-hand sides of Eq. (4) is achieved when

$$\ddot{u}_t + \omega^2 u_t = 0, \tag{5}$$

$$u''_x + \frac{S'_x}{S_x}u'_x + k^2 u_x = 0, \tag{6}$$

where $\omega$ is the ultrasound angular frequency, and $k$ represents the wave number. Furthermore we can write $\frac{d}{dx}(\ln S_x) = S'_x/S_x$.

Eq. (6) allows us to solve the problem of the shape optimization of an ultrasonic horn, with the assumption that the maximum acoustic pressure $p_{\max}$ is equal to the normal stress $\sigma_L$ at the end of the horn. We can then define a function compatible with Hooke's law and the principles of variational calculus:

$$I = \int_0^L \left[S_x + \lambda_x(S_x u''_x + S'_x u'_x + k^2 S_x u_x)\right]dx - \lambda_1\left(\frac{\sigma^2}{2E} - \frac{\sigma_L u'_L}{2}\right), \tag{7}$$

where $\lambda_x = \lambda(x)$ is a Lagrange multiplier function, $\lambda_1$ is a constant Lagrange multiplier, and $u'_L$ is the strain value at the end of the horn.

Our goal is then to find the extreme of the function $I$, i.e. to solve the Equation:

$$\delta I = 0, \tag{8}$$

with the condition $\delta u = \delta u' = 0$ at both ends of the horn [21].

Eq. (8) can be written as:

$$\delta I = \int_0^L [1 + \lambda_x(u''_x + k^2 u_x)]\delta S_x dx + \int_0^L \lambda_x(S_x \delta u''_x + u'_x \delta S'_x + S'_x \delta u'_x + k^2 S_x \delta u_x)dx - \frac{\lambda_1}{2}[u'_L \delta\sigma_L + \sigma_L \delta u'_L] \tag{9}$$

and therefore

$$\begin{cases} \lambda_x u'_x \delta S_x\big|_0^L + \lambda_x S_x \delta u'_x\big|_0^L - \frac{\lambda_1}{2}\left[u'_L\delta\sigma_L + \sigma_L \delta u'_L\right] = 0, \\ \left[(\lambda'_x S_x)' + \lambda_x k^2 S_x\right] = 0, \\ \left(1 + k^2\lambda_x u_x - \lambda'_x u'_x\right) = 0. \end{cases} \tag{10}$$

By solving the system of Eq. (10), a solution is obtained as:

$$u_x = -\frac{A}{4k^2B}e^{kx} + Be^{-kx}, \tag{11}$$