Contents lists available at ScienceDirect

Discrete Optimization

www.elsevier.com/locate/disopt

The Hamiltonian connectivity of rectangular supergrid graphs[☆]

Ruo-Wei Hung[a,*], Chin-Feng Li[a], Jong-Shin Chen[b], Qing-Song Su[c]

[a] *Department of Computer Science and Information Engineering, Chaoyang University of Technology, Wufeng, Taichung 41349, Taiwan*
[b] *Department of Information and Communication Engineering, Chaoyang University of Technology, Wufeng, Taichung 41349, Taiwan*
[c] *Department of Computer Science, Shenyang Aerospace University, 37 DaoYi South Street, DaoYi economic development Zone Shenyang City, Liaoning Province 110000, China*

A R T I C L E   I N F O

Article history:
Received 16 September 2016
Received in revised form 4 June 2017
Accepted 5 June 2017
Available online xxxx

Keywords:
Hamiltonian connectivity
Longest path
Rectangular supergrid graph
Grid graph
Computer sewing machine

A B S T R A C T

A Hamiltonian path of a graph is a simple path which visits each vertex of the graph exactly once. The Hamiltonian path problem is to determine whether a graph contains a Hamiltonian path. A graph is called Hamiltonian connected if there exists a Hamiltonian path between any two distinct vertices. In this paper, we will study the Hamiltonian connectivity of rectangular supergrid graphs. Supergrid graphs were first introduced by us and include grid graphs and triangular grid graphs as subgraphs. The Hamiltonian path problem for grid graphs and triangular grid graphs was known to be NP-complete. Recently, we have proved that the Hamiltonian path problem for supergrid graphs is also NP-complete. The Hamiltonian paths on supergrid graphs can be applied to compute the stitching traces of computer sewing machines. Rectangular supergrid graphs form a popular subclass of supergrid graphs, and they have strong structure. In this paper, we provide a constructive proof to show that rectangular supergrid graphs are Hamiltonian connected except one trivial forbidden condition. Based on the constructive proof, we present a linear-time algorithm to construct a longest path between any two given vertices in a rectangular supergrid graph.

1. Introduction

A *Hamiltonian path* in a graph is a simple path in which each vertex of the graph appears exactly once. A *Hamiltonian cycle* in a graph is a simple cycle with the same property. The *Hamiltonian path (resp., cycle) problem* involves deciding whether or not a graph contains a Hamiltonian path (resp., cycle). A graph is called *Hamiltonian* if it contains a Hamiltonian cycle. A graph G is said to be *Hamiltonian connected* if for each pair of distinct vertices u and v of G, there is a Hamiltonian path from u to v in G. If

☆ A preliminary version of this paper has appeared in: The International MultiConference of Engineers and Computer Scientists 2016 (IMECS 2016), Hong Kong, vol. I, 2016, pp. 115–121.

* Corresponding author.
E-mail address: rwhung@cyut.edu.tw (R.-W. Hung).

http://dx.doi.org/10.1016/j.disopt.2017.06.001
1572-5286/© 2017 Elsevier B.V. All rights reserved.

(u, v) is an edge of a Hamiltonian connected graph, then there exists a Hamiltonian cycle containing (u, v). Thus, a Hamiltonian connected graph has many Hamiltonian cycles, and, hence, the sufficient conditions of Hamiltonian connectivity are stronger than those of Hamiltonicity. The *longest path problem* is to find a simple path with the maximum number of vertices in a graph. The Hamiltonian path problem is clearly a special case of the longest path problem.

The Hamiltonian path and cycle problems have numerous applications in different areas, including establishing transport routes, production launching, the on-line optimization of flexible manufacturing systems [1], computing the perceptual boundaries of dot patterns [2], pattern recognition [3–5], DNA physical mapping [6], and fault-tolerant routing for 3D network-on-chip architectures [7]. It is well known that the Hamiltonian path and cycle problems are NP-complete for general graphs [8,9]. The same holds true for bipartite graphs [10], split graphs [11], circle graphs [12], undirected path graphs [13], grid graphs [14], triangular grid graphs [15], and supergrid graphs [16]. In the literature, there are many studies for the Hamiltonian connectivity of interconnection networks. Fu [17] showed the Hamiltonian connectivity of the WK-recursive network with faulty nodes. Li et al. [18] proved the Hamiltonian connectivity of the recursive dual-net. The hypercomplete network [19], the alternating group graph [20], and the arrangement graph [21] were known to be Hamiltonian connected. The popular hypercubes are Hamiltonian but are not Hamiltonian connected. However, many variants of hypercubes, including augmented hypercubes [22], generalized base-b hypercube [23], hyercube-like networks [24], twisted cubes [25], crossed cubes [26], Möbius cubes [27], folded hypercubes [28], and enhanced hypercubes [29], have been known to be Hamiltonian connected. In addition, only a few polynomial-time algorithms are known for the longest path problem on special graphs. Trees are the first class of graphs that a polynomial-time algorithm for the longest path problem has been found [30]. Uehara and Uno [31] solved the longest path problem for block graphs in linear time, for cacti in quadratic time, and for interval biconvex graphs in polynomial time. Ioannidou et al. [32] showed that the longest path problem on interval graphs is polynomially solvable. Mertzios and Corneil [33] solved the problem in polynomial time for a larger class of graphs, named cocomparability graphs. More recently, Mertzios and Bezáková [34] solved the problem on circular-arc graphs in polynomial time. Also, there is a linear-time algorithm for the longest path problem on rectangular grid graphs proposed by Keshavarz-Kohjerdi et al. [35].

The *two-dimensional integer grid* G^∞ is an infinite graph whose vertex set consists of all points of the Euclidean plane with integer coordinates and in which two vertices are adjacent if the (Euclidean) distance between them is equal to 1. A *grid graph* is a finite, vertex-induced subgraph of G^∞. For a node v in the plane with integer coordinates, let v_x and v_y represent the x and y *coordinates* of node v, respectively, denoted by $v = (v_x, v_y)$. If v is a vertex in a grid graph, then its possible adjacent vertices include $(v_x, v_y - 1)$, $(v_x - 1, v_y)$, $(v_x + 1, v_y)$, and $(v_x, v_y + 1)$. The *two-dimensional triangular grid* T^∞ is an infinite graph obtained from G^∞ by adding all edges on the lines traced from up-left to down-right. A *triangular grid graph* is a finite, vertex-induced subgraph of T^∞. If v is a vertex in a triangular grid graph, then its possible neighboring vertices include $(v_x, v_y - 1)$, $(v_x - 1, v_y)$, $(v_x + 1, v_y)$, $(v_x, v_y + 1)$, $(v_x - 1, v_y - 1)$, and $(v_x + 1, v_y + 1)$. Thus, triangular grid graphs contain grid graphs as subgraphs. Note that triangular grid graphs defined above are isomorphic to the original triangular grid graphs studied in the literature [15] but these graphs are different when considered as geometric graphs. By the same construction of triangular grid graphs from grid graphs, we have proposed a new class of graphs, namely *supergrid graphs*, in [16]. The *two-dimensional supergrid* S^∞ is an infinite graph obtained from T^∞ by adding all edges on the lines traced from up-right to down-left. A *supergrid graph* is a finite, vertex-induced subgraph of S^∞. The possible adjacent vertices of a vertex $v = (v_x, v_y)$ in a supergrid graph include $(v_x, v_y - 1)$, $(v_x - 1, v_y)$, $(v_x + 1, v_y)$, $(v_x, v_y + 1)$, $(v_x - 1, v_y - 1)$, $(v_x + 1, v_y + 1)$, $(v_x + 1, v_y - 1)$, and $(v_x - 1, v_y + 1)$. For example, Fig. 1(a)–(c) show a grid graph, a triangular grid graph, and a supergrid graph, respectively. Then, supergrid graphs contain grid graphs and triangular grid graphs as subgraphs. Notice that grid and triangular grid graphs are not subclasses of supergrid graphs, and the converse is also true: these classes of graphs have common elements (vertices) but in general they

Please cite this article in press as: R.-W. Hung, et al., The Hamiltonian connectivity of rectangular supergrid graphs, Discrete Optimization (2017), http://dx.doi.org/10.1016/j.disopt.2017.06.001.