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Efficient approximation schemes for Economic Lot-Sizing in continuous time

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We consider a continuous-time variant of the classical Economic Lot-Sizing (**ELS**) problem. In this variant, the setup cost is a continuous function with lower bound $K_{\min} > 0$, the demand and holding costs are integrable functions of time and arbitrary replenishment policies are allowed. Starting from the assumption that certain operations involving the setup and holding cost functions can be carried out efficiently, we show that this variant admits a simple approximation scheme based on dynamic programming: if the optimal cost of an instance is **OPT**, we can find a solution with cost at most $(1+\epsilon)\mathbf{OPT}$ using no more than $O\left(\frac{1}{\epsilon^2}\frac{\mathbf{OPT}}{K_{\min}}\log\frac{\mathbf{OPT}}{K_{\min}}\right)$ of these operations. We argue, however, that this algorithm could be improved on instances where the setup costs are generally "very large" compared with $K_{\min}$. This leads us to introduce a notion of input-size parameter σ that is significantly smaller than $\mathbf{OPT}/K_{\min}$ on instances of this type, and then to define an approximation scheme that executes $O\left(\frac{1}{\epsilon^2}\sigma^2\log^2\left(\frac{\mathbf{OPT}}{K_{\min}}\right)\right)$ operations. Besides dynamic programming, this second approximation scheme builds on a novel algorithmic approach for Economic Lot Sizing problems.

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1. Introduction

Economic Order Quantity (**EOQ**) and Economic Lot-Sizing (**ELS**) are two classical problems in inventory management [1–3]. In both problems the overall cost decomposes into a fixed ordering/production cost that is charged each time there is a production order, and the holding costs that are charged proportional to the stock level. The main trade-off in **EOQ** and **ELS** is between low setup costs (few batches) and low holding costs (many batches or just-in-time production). This captures a key aspect of most single-commodity production planning problems. The two problems differ by other features: **EOQ** assumes constant cost

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functions and demand rate on a continuous-time infinite time horizon and admits an analytical solution [4, Ch. 2 Sec. 2] while **ELS** assumes time-varying costs and demand in a finite discretized planning horizon and is solved by dynamic programming [3].

In this work, we consider the natural generalization of both **EOQ** and **ELS**: we handle time-varying cost and demand rates in continuous time in a finite planning horizon. In practice, considering decisions in continuous time is becoming more and more realistic as production processes become fully automated. Moreover, in industrial context, discretization of time in **ELS** is typically made before input data is known. In a first stage, the granularity of the discretization is determined based on general information about the type of problem, the reliability of input data or the planning horizon, among others. In a second stage, the precise input data is determined and the problem solved. As will appear below, our algorithm considers only a finite set of possible times for placing setups, implicitly building a discretization of the time horizon. Building a good discretization of the time horizon is part of the decisions made by our algorithm. Therefore, one can view the problem treated here as the integration of two optimization problems that are usually solved sequentially: discretization of time and timing of production.

Finally, another view is that the problem we consider is equivalent to **ELS** when n the number of time periods becomes very large and already $O(n)$ time complexity is prohibitive. We will show that under mild conditions on the input data and the way it is accessible, we can obtain a provably good solution in $O(\log n)$ time. Therefore our work fits into the rapidly expanding field of sub-linear time algorithms, see [5].

1.1. Formal problem description

Formally, the input of the problem we consider is given by the following objects: an integrable *demand function* $d : [0, T] \to \mathbb{R}^+$, an integrable *holding cost function* $h : [0, T] \to \mathbb{R}^+$, and a continuous *setup cost function* $K : [0, T] \to [K_{\min}, K_{\max}]$ with a strictly positive lower bound $K_{\min} > 0$. The interval $[0, T]$ is called the *planning horizon.* A feasible solution for this problem is any finite sequence of strictly increasing setups $\mathbf{s} \equiv \{s_i\}_{i=1..n} \subseteq [0, T]$ with $s_1 = 0$; for notational convenience, $s_{n+1} = T$ is defined, but it will not be accounted as a setup (by setting $K_{n+1} = 0$). Because h is non-negative, given the setup sequence it is always optimal to produce as late as possible (i.e. a zero-inventory policy is optimal). Therefore the setup times implicitly partition the time horizon into n *windows* $[s_i, s_{i+1})$, where the inventory required to fulfill demands in $[s_i, s_{i+1})$ is defined to be ordered at s_i. The order at s_i incurs a holding cost of

$$H_i \equiv \int_{s_i}^{s_{i+1}} h(t) D_{\mathbf{s}}(t)dt, \quad \text{where } D_{\mathbf{s}}(t) = \int_t^{s_{i+1}} d(u)du, \text{ for } t \in [s_i, s_{i+1}]$$

represents the stored demand at time t. The order also incurs a setup cost of

$$K_i \equiv K(s_i).$$

Note that for both, K_i and H_i, the dependency on $\mathbf{s}$ is omitted, and will be normally inferred from the context. The objective of the **ELS** problem is to find a feasible solution $\{s_i\}_{i=1..n}$ minimizing the total cost $\sum_{i=1}^{n} (K_i + H_i)$. The restriction to $K(\cdot) \geq K_{\min} > 0$ forces near optimal solutions (i.e. solutions whose cost approximates OPT within a constant factor) to have a bounded number of setups; hence by compactness the global minimum for the **ELS** problem is guaranteed to exist. In the remainder of the paper, we will assume without loss of generality that $K_{\min} = 1$.

1.2. Previous work

Massonnet [6] gives an approximation guarantee for the continuous-time **ELS** problem with general time-varying demand rate. Assuming a constant setup cost function K, he shows that balancing setup and

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