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An exact frequency analysis of annular plates with small core having elastically restrained outer edge and sliding inner edge

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A B S T R A C T

The present study deals with an exact analysis of free transverse vibrations of annular plates having small core and sliding inner edge and the outer edge being elastically restrained based on classical plate theory. This study focuses mainly on the influence of variations in the elastic restraint parameters on the fundamental frequencies of plate vibration. The natural frequencies for the first six modes of annular plate vibrations are computed for different materials and varying values of the radius parameter and these natural frequencies may correspond to either axisymmetric and/or non-axisymmetric modes of plate vibration. The extensive data of values of fundamental frequency parameter presented in this paper is believed to be of use in the design of acoustic underwater transducers, ocean and naval structures, compressor and pump elements, offshore platforms. These results may serve as bench mark values for researchers to validate their results obtained using approximate numerical methods.

1. Introduction

Vibrations of annular plates are an important area of interest in the design of commonly used structural components in the aerospace and nuclear plant structures. There exists a considerable amount of literature on the vibration of annular plates, especially with classical edge conditions [1–4]. The fundamental frequency corresponds in general to the axisymmetric mode with no nodal diameter, and the plate with both edges free vibrates with two nodal diameters. Interestingly, the fundamental frequency may switch from no nodal diameter to one as the core radius is decreased for annular plates with their inner edge being either clamped or simply supported and the outer edge being free [5–10].

Southwell [11] analytically studied the problem of free vibration characteristics of clamped–free annular plates by employing the method of asymptotic expansions the case of the radial distance b approaches zero. It is found that the frequency rises singularly from zero for smaller values of b. Kim and Dickinson [12] studied the influence of elastic edge restraints on the natural frequencies of polar orthotropic and isotropic annular and circular plates. Vera et al. [13,16] investigated the effect of elastic restraint against rotation on the frequencies of vibration of circular annular plates which has practical application in acoustic underwater transducers. Recently Rao and Rao [14] studied the problem of buckling of annular plates involving elastic restraints and sliding ends. Further Rao and Rao [15] studied the vibration characteristics of circular plates resting on elastic foundation involving sliding edge conditions. The free vibration characteristics of circular plates with attached core are studied firstly by Wang and Wang [16]. In a more recent study, Yuce and Wang [17] discussed and presented perturbation methods for the parallel case of a moderately elliptical plate with an attached core.

The case of annular plates with small core was analyzed by Wang [18] and they presented results for the fundamental natural frequencies especially for the case of non-dimensional radial distance b less than 0.1. However, their study deals with annular plates having combinations of classical boundary conditions such as clamped, simply supported or free boundary conditions at the inner and outer edges of the plate. As we know in practical industrial applications of annular plates they are often subjected to complex combinations of classical boundary conditions more closely modeled using elastic restraints such as rotational and translational restraints [19,11–15].

To circumvent the difficulty of solving transcendental frequency equations that result from exact methods of solution which involve trigonometric, hyperbolic or Bessel functions, Li [20,21] developed a modified Fourier series technique and applied the same successfully to solve the free vibration problem of generally restrained

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beams. Based on the excellent convergence obtained from the modified Fourier series method together with Ritz method, it is widely used in various studies on vibrations of functionally graded circular, annular and sector plates and curved shells involving various complex boundary conditions [22–30].

Using exact analysis methodology, the purpose of the present paper is to investigate, the influence of edge restraints on the fundamental frequencies of the annular plates with small core and having sliding inner edge and outer edge being elastically restrained. The accurate and exact natural frequency data presented in this study is believed to be of use in improving the performance of certain economic underwater acoustic transducers and the design of parts of ship decks to printed circuit boards passing through nuclear reactor elements and mechanical applications in engines, etc.

2. Mathematical formulation

Let us consider a thin circular annular plate of outer radius R, inner radius bR, uniform thickness h, Young's modulus E, flexural rigidity D and Poisson's ratio ν. The annular plate is also assumed to be made of linearly elastic, homogeneous and isotropic material. The outer edge of the annular plate is elastically restrained against rotation and translation and inner edge is sliding. Let the symbol I denote the outer region $b \leqslant r \leqslant 1$ and II denote the inner region $0 \leqslant r \leqslant b$. Here all lengths are normalized with respect to R, i.e., the radius of outer region is 1 and that of inner region is b (see Fig. 1).

In the classical plate theory [1], the following fourth order differential equation describes free transverse vibrations of a thin circular uniform plate.

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

where $D = Eh^3/12(1-\nu^2)$. The general form of the lateral displacement of the vibration of a classical thin plate can be expressed as $w = u(r)\cos(n\theta)e^{i\Omega t}$, where w is the transverse displacement, n is the integer and Ω is the frequency. The function $u(r)$ is a linear combination of the Bessel functions $J_n(kr)$, $Y_n(kr)$, $I_n(kr)$ and $K_n(kr)$. Where $k = R\left(\frac{\rho h \Omega^2}{D}\right)^{1/4}$ is the square root of the non-dimensional frequency [1].

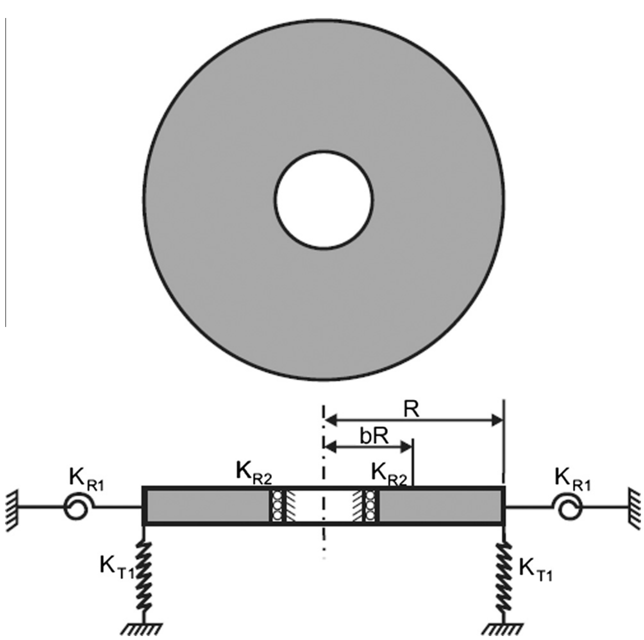

Fig. 1. Annular plate with small core (RT-G boundary).

Following the exact method of analysis, the general solution of Eq. (1) is given by

$$u(r) = C_1 J_n(kr) + C_2 Y_n(kr) + C_3 I_n(kr) + C_4 K_n(kr) \tag{2}$$

Case (i) Outer edges of an annular plate is elastically retrained against rotation and translation:

The boundary conditions at outer region of the annular plate can be formulated in terms of effective rotational (K_{R1}) and translational (K_{T1}) stiffness

$$M_r(r,\theta) = K_{R1}\frac{\partial w_I(r,\theta)}{\partial r} \tag{3}$$

$$V_r(r,\theta) = -K_{T1} w_I(r,\theta) \tag{4}$$

where the bending moment and the Kelvin–Kirchhoff shear force are defined as follows

$$M_r(r,\theta) = -\frac{D}{R}\left[\frac{\partial^2 w_I(r,\theta)}{\partial r^2} + \nu\left(\frac{1}{r}\frac{\partial w_I(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w_I(r,\theta)}{\partial \theta^2}\right)\right] \tag{5}$$

$$V_r(r,\theta) = -\frac{D}{R^3}\left[\frac{\partial}{\partial r}\nabla^2 w_I(r,\theta) + (1-\nu)\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{1}{r}\frac{\partial^2 w_I(r,\theta)}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial w_I(r,\theta)}{\partial\theta}\right)\right] \tag{6}$$

Eqs. (3) & (5) and (4) & (6) can be written as

$$\left[\frac{\partial^2 w_I(r,\theta)}{\partial r^2} + \nu\left(\frac{1}{r}\frac{\partial w_I(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w_I(r,\theta)}{\partial\theta^2}\right)\right] = -R_{11}\frac{\partial w_I(r,\theta)}{\partial r} \tag{7}$$

$$\left[\frac{\partial}{\partial r}\nabla^2 w_I(r,\theta) + (1-\nu)\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{1}{r}\frac{\partial^2 w_I(r,\theta)}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial w_I(r,\theta)}{\partial\theta}\right)\right] = T_{11} w_I(r,\theta) \tag{8}$$

where $R_{11} = \frac{K_{R1}R}{D}$ and $T_{11} = \frac{K_{T1}R^3}{D}$ are normalized spring constants of K_{R1} and K_{T1} of the rotational and translational elastic springs respectively.

At $r = 1$, the Eqs. (7) and (8) can be written as

$$u''(1) + \nu(u'(1) - n^2 u(1)) = -R_{11}u'(1) \tag{9}$$

$$u'''(1) + u''(1) - [1 + n^2(2-\nu)]u'(1) + n^2(3-\nu)u'(1)) = T_{11}u(1) \tag{10}$$

Case (ii) Inner edge of an annular plate is sliding:

The boundary conditions at the inner region of the annular plate can be formulated as:

$$\frac{\partial w_{II}(r,\theta)}{\partial r} = 0 \tag{11}$$

$$V_r(r,\theta) = 0 \tag{12}$$

where the Kelvin–Kirchhoff shear force is defined as follows

$$V_r(r,\theta) = -\frac{D}{R^3}\left[\frac{\partial}{\partial r}\nabla^2 w_{II}(r,\theta) + (1-\nu)\frac{1}{r}\frac{\partial}{\partial\theta} \times \left(\frac{1}{r}\frac{\partial^2 w_{II}(r,\theta)}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial w_{II}(r,\theta)}{\partial\theta}\right)\right] \tag{13}$$

Eqs. (12) and (13) can be written as