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PII: S0167-6377(17)30516-3
DOI: https://doi.org/10.1016/j.orl.2018.07.004
Reference: OPERES 6380

To appear in: *Operations Research Letters*

Received date : 9 September 2017
Revised date : 9 July 2018
Accepted date : 10 July 2018

Please cite this article as: M. Shahabsafa, J.C. Góez, T. Terlaky, On pathological disjunctions and redundant disjunctive conic cuts, *Operations Research Letters* (2018), https://doi.org/10.1016/j.orl.2018.07.004

On Pathological Disjunctions and Redundant Disjunctive Conic Cuts

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Abstract

The development of Disjunctive Conic Cuts (DCCs) for Mixed Integer Second Order Cone Optimization (MISOCO) problems has recently gained significant interest in the optimization community. In this paper, we explore the pathological disjunctions where disjunctive cuts do not tighten the description of the feasible set. We focus on the identification of cases when the generated DCCs are redundant. Avoiding the generation of redundant cuts saves computational time and facilitates efficient implementation of branch and cut algorithms.

Keywords: Conic Optimization, Disjunctive Conic Cuts, Mixed Integer Second Order Cone Optimization
2010 MSC: 00-01, 99-00

1. Introduction

A Mixed Integer Second Order Cone Optimization (MISOCO) problem may be formulated as

$$\begin{aligned} \min \quad & c^\top x \\ \text{s.t.} \quad & Ax = b \\ & x \in \mathcal{L} \\ & x \in \mathbb{Z}^d \times \mathbb{R}^{n-d}, \end{aligned} \tag{1}$$

in which $c \in \mathbb{R}^n$, $b \in \mathbb{R}^m$, $A \in \mathbb{R}^{m\times n}$ and $\mathcal{L} = \mathcal{L}_1 \times \mathcal{L}_2 \times \cdots \times \mathcal{L}_k$ is the Cartesian product of SOCs, each of which is defined as follows

$$\mathcal{L}_i = \{(x_0^i, x^i) \in \mathbb{R} \times \mathbb{R}^{n_i-1} \mid \|x^i\|_2 \le x_0^i\}, \quad i = 1, \ldots, k,$$

with $\sum_{i=1}^k n_i = n$.

In principle, one can solve a MISOCO problem exactly using a branch and cut methodology. One of the key elements of branch and cut is the derivation of effective and efficient cuts to strengthen the formulation. Recent studies have shown the performance improvements that one can obtain in a branch and cut using the lift-and-project approach, which can significantly reduce the solution time [16]. Most of the work on cut generation has been focused on obtaining valid linear inequalities. However, the possibility of generating nonlinear cuts for MISOCO problems have recently received significant attention in the optimization community.

Stubbs and Mehrotra [22] extended Balas et al. [6] lift and project procedure to 0-1 mixed integer convex optimization problems. They derive valid inequalities for mixed integer problems by solving a convex optimization sub-problem. Çezik and Iyengar [11] derive convex cuts for mixed 0-1 conic optimization problems. They consider the linear cone, the second-order cone and the cone of positive semidefinite matrices and extend a variety of techniques, used in generating cuts for MILO problems such as Gomory cuts [15] and lift and project cuts.

For MISOCO problems in particular, Atamtürk and Narayanan [3, 4] extended the idea of mixed integer rounding cuts developed by Nemhauser and Wolsey [19]. They reformulated a SOC in terms of two-dimensional polyhedral SOCs and designed a rounding procedure to derive conic cuts for the original MISOCO problem. Kılınç-Karzan and Yıldız [17] consider a two-term disjunction on a SOC and derive closed-form convex inequalities describing the convex hull of the intersection of the disjunction with the cone. They characterize the cases where one SOC inequality is enough to describe the mentioned convex hull. Drewes [13] presents lift-and-project based linear and convex quadratic cuts for mixed 0-1 SOCO problems. Dadush et al. [12] extend the idea of split cuts [19] for a full dimensional ellipsoid. They consider parallel disjunctions on ellipsoids and derive a conic cut which describes the convex hull of the ellipsoid intersected with the disjunctive set. Andersen and Jensen [2] extend the idea of intersection cuts [5] to mixed integer conic quadratic sets. Modaresi et al. [18] explains the relationship between mixed integer rounding cuts [3, 4] and split cuts, [12] and discusses the trade-off between computational cost of adding the split cuts and strength of the formulation resulted from adding them to the model.

Belotti et al. [7, 8, 9] consider a disjunction on a general MISOCO problem and generate a class of cuts called *Disjunctive Conic Cuts (DCCs)* and *Disjunctive Cylindrical Cuts (DCyC)*. The DCCs and DCyCs describe the convex hull of the intersection of the disjunction with the feasible set of the continuous relaxation of a MISOCO problem. In other words, the intersection of the DCC with the feasible set of the continuous relax-

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