The multiobjective steepest descent direction is not Lipschitz continuous, but is Hölder continuous

Benar F. Svaiter

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Benar F. Svaiter* †

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Abstract

The aim of this manuscript is to characterize the continuity properties of the multiobjective steepest descent direction (vector field) for smooth objective functions. We will show that this vector field is Hölder continuous with optimal exponent 1/2. In particular, this vector field fails to be Lipschitz continuous even for polynomial objectives.

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Key words: multiobjective optimization; steepest descent direction; Lipschitz continuity; Hölder continuity

multiobjective optimization; Pareto optimality, steepest descent direction, Lipschitz continuity, Hölder continuity.

1 Introduction

In multiobjective optimization (MO) problems many functions on the same argument have to be simultaneously minimized. Since in general there is no common minimizer for these functions, one shall rely on another notion of optimality. Up to now, in this setting, the most useful definition of optimality is that of Pareto [11]: A point is a Pareto optimal if its image is minimal in the image of the feasible set with respect to the componentwise (partial) order. Vector optimization is a generalization of MO where a closed convex cone is used to define the partial order for which minimal elements are to be computed. If the cone is the positive orthant one retrieves the componentwise order of MO.

Descent methods for multiobjective and vector optimization are, presently, areas of intense research (see [5, 3, 2, 4, 13, 9, 16, 8, 14, 12, 15, 7, 1, 6] and the references therein). These are iterative methods in which *all* objective functions decrease along the generated sequences. As far as we now know, the first one of these methods was proposed by Mukai in [10]. In this work, six descent methods for *constrained* MO were proposed, three of which become, in the unconstrained case, the steepest descent method. In his pioneer work, Mukai neither studied the continuity properties of the MO

*IMPA, Estrada Dona Castorina 110, 22460-320 Rio de Janeiro, Brazil (benar@impa.br) tel: 55 (21) 25295112, fax: 55 (21)25124115.

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