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Some theoretical limitations of second-order algorithms for smooth constrained optimization

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Abstract

In second-order algorithms, we investigate the relevance of the constant rank of the full set of active constraints in ensuring global convergence to a second-order stationary point under a constraint qualification. We show that second-order stationarity is not expected in the non-constant rank case if the growth of so-called tangent AKKT2 sequences is not controlled. Since no algorithm controls their growth, we argue that there is a theoretical limitation of algorithms in finding second-order stationary points without constant rank assumptions.
Keywords: Global convergence, Second-order algorithms, Constant rank, Constraint qualification, Second-order optimality conditions

1 Introduction

In this paper we are interested in optimality properties of limit points of sequences generated by numerical algorithms, that is, global convergence results. Let us assume that x^* is a feasible limit point of a sequence $\{x^k\}$ generated by an unspecified first-order numerical algorithm. It is well known that assuming that x^* satisfies some constraint qualification is not enough to ensure that x^* is a Karush-Kuhn-Tucker (KKT) point. Some specific constraint qualification, what has been called a strict one in [4], is needed. Similarly, if the algorithm is a second-order one, the Weak Second-order Optimality Condition (WSOC), the standard necessary optimality condition based on the critical subspace, can usually be proved at x^* under a constraint qualification stronger than what is needed to prove that WSOC is necessary at a local minimizer (see [8]).

In this sense, there is an intrinsic theoretical gap between the types of problems in which local solutions are known to be KKT or WSOC points, and the attainability of such points by a numerical algorithm. There is also a gap in the sense that the second-order optimality condition can be theoreticaly refined to consider a larger set of directions, namely, the critical cone, in which the critical

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