

## Accepted Manuscript

Spitzer's identity for discrete random walks

A.J.E.M. Janssen, J.S.H. van Leeuwaarden

PII: S0167-6377(17)30592-8  
DOI: <https://doi.org/10.1016/j.orl.2017.12.003>  
Reference: OPERES 6317

To appear in: *Operations Research Letters*

Received date: 25 October 2017  
Revised date: 6 December 2017  
Accepted date: 10 December 2017



Please cite this article as: A.J.E.M. Janssen, J.S.H. van Leeuwaarden, Spitzer's identity for discrete random walks, *Operations Research Letters* (2017), <https://doi.org/10.1016/j.orl.2017.12.003>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

# Spitzer's identity for discrete random walks \*

A.J.E.M. Janssen <sup>†</sup>

J.S.H. van Leeuwaarden <sup>‡</sup>

December 6, 2017

## Abstract

Spitzer's identity describes the position of a reflected random walk over time in terms of a bivariate transform. Among its many applications in probability theory are congestion levels in queues and random walkers in physics. We present a derivation of Spitzer's identity for random walks with bounded jumps to the left, only using basic properties of analytic functions and contour integration. The main novelty is a reversed approach that recognizes a factored polynomial expression as the outcome of Cauchy's formula.

**Keywords:** Spitzer's identity; fluctuation theory; transform methods; complex analysis

## 1 Introduction

Random walks are ubiquitous in the modern stochastics literature. This paper deals with the one-dimensional random walk on  $\mathbb{Z}$  describing the partial sums  $S_n := X_1 + \dots + X_n$  ( $S_0 := 0$ ) of i.i.d. random variables  $X_1, X_2, \dots$ . The stochastic process  $(S_n, n \geq 0)$ , the random walk with steps  $X_n$ , arises in many areas of science to describe the evolution of certain objects subject to random fluctuations, including random walkers in physics, congestion levels in queueing theory and capital positions in insurance mathematics [9, 13]. If we indeed think of a random walk  $(S_n, n \geq 0)$  as modeling capital or congestion, the monetary/physical interpretation means that large values of  $S_n$  are of particular interest, and it is natural to consider the sequence  $\bar{M}_n := \max\{S_0, S_1, \dots, S_n\}$ . The study of  $(\bar{M}_n, n \geq 0)$  and related quantities is referred to as the fluctuation theory of the random walk, a central topic in most classic probability textbooks [3, 7, 8, 12, 21].

Fluctuation theory became highly topical by the rise of queueing theory in the first half of the twentieth century, with foundational works of A.K. Erlang and F. Pollaczek (see the historical account in [14]) and as primary example the waiting time process in the single-server queue. Let consecutive customers arriving to a single-server queue be numbered  $n = 1, 2, \dots$ . Denote by  $B_n$  the service time of customer  $n$ , and by  $C_n$  the time between the arrivals of customers  $n$  and  $n + 1$ . Then with  $X_{n+1} := B_n - C_n$ , and  $W_n$  the waiting time of customer  $n$ , the waiting time process (also known as the Lindley process) is given by

$$W_{n+1} = (W_n + X_{n+1})^+, \quad n = 0, 1, \dots, \quad (1.1)$$

\*Dedicated to Prof. Onno J. Boxma on the occasion of his 65th birthday

<sup>†</sup>Department of Mathematics and Computer Science, Eindhoven University of Technology, The Netherlands

<sup>‡</sup>Department of Mathematics and Computer Science, Eindhoven University of Technology, The Netherlands. Email: j.s.h.v.leeuwaarden@tue.nl (corresponding author)

Download English Version:

<https://daneshyari.com/en/article/7543831>

Download Persian Version:

<https://daneshyari.com/article/7543831>

[Daneshyari.com](https://daneshyari.com)