Accepted Manuscript

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PII: S0167-6377(17)30338-3
DOI: https://doi.org/10.1016/j.orl.2017.10.011
Reference: OPERES 6289

To appear in: *Operations Research Letters*

Received date : 15 June 2017
Accepted date : 17 October 2017

Please cite this article as: H. Gerard, V. Leclere, A. Philpott, On risk averse competitive equilibrium, *Operations Research Letters* (2017), https://doi.org/10.1016/j.orl.2017.10.011

On risk averse competitive equilibrium

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Abstract

We discuss risked competitive partial equilibrium in a setting in which agents are endowed with coherent risk measures. In contrast to social planning models, we show by example that risked equilibria are not unique, even when agents' objective functions are strictly concave. We also show that standard computational methods find only a subset of the equilibria, even with multiple starting points.

Keywords: Stochastic Equilibrium, Stochastic Programming, Risk averse equilibrium, Electricity Markets

1. Introduction

Most industrialised regions of the world have over the last thirty years established wholesale electricity markets that take the form of an auction that matches supply and demand. The exact form of these auction mechanisms vary by jurisdiction, but they typically require offers of energy from suppliers at costs they are willing to supply, and clear a market by dispatching these offers in order of increasing cost. Day-ahead markets such as those implemented in many North American electricity systems, seek to arrange supply well in advance of its demand, so that thermal units can be prepared in time. Since the demand cannot be predicted with absolute certainty, day-ahead markets must be accompanied by a separate balancing market to deal with the variation in load and generator availability in real time. These are often called *two-settlement* markets. The market mechanisms are designed to be as efficient as possible in the sense that they should aim to maximize the total welfare of producers and consumers.

In response to pressure to reduce CO_2 emissions and increase the penetration of renewables, electricity pool markets are procuring increasing amounts of electricity from intermittent sources such as wind and solar. If probability distributions for intermittent supply are known for these systems then it makes sense to maximize the expected total welfare of producers and consumers in each dispatch. Then many repetitions of this will yield a long run total benefit that is maximized. Maximizing expected welfare can be modeled as a two-stage stochastic program. Methods for computing prices and single-settlement payment mechanisms for such a *stochastic market clearing* mechanism are described in a number of papers (see Pritchard et al. [11], Wong and Fuller [16] and Zakeri et al. [17]). When evaluated using the assumed probability distribution on supply, stochastic market clearing can be shown to be more efficient than two-settlement systems.

If agents in these systems are risk averse then one might also seek to maximize some risk-adjusted social welfare. In this setting the computation of prices and payments to the agents becomes more complicated. If agents use coherent risk measures then it is possible to define a complete market for risk in a precise sense. If the market is complete then a perfectly competitive partial equilibrium will also maximize risk-adjusted social welfare, i.e. it is efficient. On the other hand if the market for risk is not complete, then perfectly competitive partial equilibrium can be inefficient. This has been explored in a number of papers (see e.g. de Maere d'Aertrycke et al. [4], Ehrenmann and Smeers [5] and Ralph and Smeers [12]).

In this paper we study a class of stochastic dispatch and pricing mechanisms under the assumption that agents will attempt to maximize their risk-adjusted welfare at these prices. Agents have coherent risk measures and are assumed to behave as price takers in the energy and risk markets. We aim at enlightening some difficulties that arise when risk markets are not complete. We describe a simple instance of a stochastic market that has three different equilibria. Two of these points are stable in the sense of Samuelson [13] and are attractors of tatônnement algorithms. The third equilibrium is unstable, yet is the solution yielded by the well-known PATH solver in GAMS (See Ferris and Munson [8]). Our example illustrates the delicacy of seeking numerical solutions for equilibria in incomplete markets. Since these are used for justifying decisions, the nonuniqueness of solutions in this setting is undesirable.

The paper is laid out as follow. In Section 2 we present the equilibrium and optimization models we are going to study. In Section 3 we give links between equilibrium and optimization problems in the risk neutral and complete risk-averse cases. Finally, in Section 4 we showcase a simple example with multiple equilibria in the incomplete risk-averse case.

1.1. Notation

We use the following notation throughout the paper: $[\![a;b]\!]$ is the set of integers between a and b (included), random variables are denoted in bold, Ω is a finite sample space, $\mathbb{P}$ is a probability distribution over Ω, $\mathbb{E}_{\mathbb{P}}$ is used to refer to expectation with