Accepted Manuscript

Optimality of an affine intensity policy for maximizing the probability of an arrival count in point-process intensity control

Boris Defourny

PII: S0167-6377(17)30485-6
DOI: https://doi.org/10.1016/j.orl.2017.11.003
Reference: OPERES 6299

To appear in: *Operations Research Letters*

Received date : 1 September 2017
Revised date : 11 October 2017
Accepted date : 3 November 2017

Please cite this article as: B. Defourny, Optimality of an affine intensity policy for maximizing the probability of an arrival count in point-process intensity control, *Operations Research Letters* (2017), https://doi.org/10.1016/j.orl.2017.11.003

Optimality of an Affine Intensity Policy for Maximizing the Probability of an Arrival Count in Point-Process Intensity Control

Boris Defourny[1]

Department of Industrial and Systems Engineering, Lehigh University, 200 W Packer Ave, Bethlehem, PA, 18015, USA

Abstract

This paper considers the problem of maximizing the probability of attaining a prescribed count of arrivals generated by a point process, by controlling its intensity. Our analysis shows the existence of optimal intensity switching times that are affine in the arrival count, thereby contributing to the literature on the optimality of affine policies. The optimal intensity control law is established, along with closed-form expressions for its numerical parameters. Several properties of the value function are listed as well.

Keywords: Point processes, Optimal stochastic control, Intensity control, Affine decision rules

---

1. Introduction

This paper considers the problem of controlling the intensity of a point process in order to maximize the probability that a target number of arrivals is met exactly by a deadline, assuming the intensity is allowed to vary within a finite range [1]. The point process is assumed to be a simple point process, that is, arrivals happen one at a time. Mathematically, the problem can be formulated as an optimal point-process intensity control problem,

$$\begin{aligned} V &= \max_{\pi} \mathbb{P}^{\pi}\left[S_T = C | S_0 = 0\right] \\ &= \max_{\pi} \mathbb{E}^{\pi}\left[R_T(S_T) | S_0 = 0\right], \end{aligned} \tag{1}$$

where the state S_t represents the count of arrivals during the time period $(0, t]$, following a point process with controlled intensity

$$\lambda_t = \lambda_t^{\pi}(S_t) \in [\lambda_a, \lambda_b], \qquad 0 < \lambda_a < \lambda_b < \infty. \tag{2}$$

The terminal reward function is defined as

$$R_T(S_T) = 1_C(S_T) \tag{3}$$

which is equal to 1 if $S_T = C$ and 0 otherwise. The control law λ_t^{π} to be optimized is a function of the state $S_t \in \mathbb{N}$ and of the time $t \in [0, T]$.

The number of arrivals during a small time interval $(t, t + dt]$ follows a Poisson distribution of mean $\lambda_t dt$. At the first-order, $\mathbb{P}(S_{t+dt} - S_t = 1) = \lambda_t dt + o(dt)$, $\mathbb{P}(S_{t+dt} - S_t \geq 2) = o(dt)$, and $\mathbb{P}(S_{t+dt} - S_t = 0) = (1 - \lambda_t dt) + o(dt)$ [2]. Thus, while λ_t is controlled, there is no direct control over the arrival times (since $\lambda_b < \infty$).

Our interest in (1) stems from the fact that it represents one of the simplest possible point-process control problems where the decision is the intensity, and yet its optimal solution has not been described satisfactorily. What is known is the existence of an optimal bang-bang intensity policy [1, 3]. The Hamilton-Jacobi-Bellman equations characterizing the optimal solution to the continuous-time, infinite-dimensional control problem are known [4]. As a result, solutions to discretized versions of the problem can be obtained via numerical solution algorithms, developed for instance in [5]. An infinite-dimensional, nonconvex formulation for policy optimization is known as well [1].

While numerical approaches are applicable, there is value in pursuing the analysis of the optimal control problem further. Additional insights facilitate sensitivity and robustness studies and suggest approaches to tackle higher-dimensional problems. Point-process control has applications in areas such as queueing systems, inventory control, and revenue management. There is often a direct relationship between intensities and prices – for instance, [6] hypothesizes a relationship between prices and rate of arrival of customers to optimize a price-setting policy for selling an inventory of perishable products. In [7], the authors hypothesize that productivity is optimized by receiving the right workload,

---

[1] Corresponding author. Email address: defourny@lehigh.edu