Accepted Manuscript

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PII: S0167-6377(17)30107-4
DOI: https://doi.org/10.1016/j.orl.2017.11.007
Reference: OPERES 6303

To appear in: *Operations Research Letters*

Received date : 21 February 2017
Revised date : 9 November 2017
Accepted date : 9 November 2017

Please cite this article as: T. Harks, M. Klimm, M. Schneider, Bottleneck routing with elastic demands, *Operations Research Letters* (2017), https://doi.org/10.1016/j.orl.2017.11.007

Bottleneck Routing with Elastic Demands☆

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Abstract

Bottleneck routing games are a well-studied model to investigate the impact of selfish behavior in communication networks. In this model, each user selects a path in a network for routing her *fixed* demand. The disutility of a user only depends on the most congested link visited. We extend this model by allowing users to continuously vary the demand rate at which data is sent along the chosen path. As our main result we establish tight conditions for the existence of pure strategy Nash equilibria.

Keywords: Bottleneck routing game, Elastic demands, Pure Nash equilibrium

1. Introduction

Bottleneck routing games are a theoretical model to study the effects of resource allocation in distributed communication networks [1, 4]. Every user of the network is associated with a non-negative demand that she wants to send from her source to the respective destination, and her goal is to find a path that minimizes the congestion of the most congested link. It has been argued (cf. [5, 28]) that in the context of packet-switched communication networks, the performance of a path is closer related to the most congested link as compared to the classical sum-aggregation of costs (as in [18, 29, 34]), and there are several proposals (cf. [26, 36]) for replacing the sum-aggregation of congestion costs with the max-aggregation, primary, because the max-aggregation leads to favorable properties of protocols in terms of their stability in presence of communication delays [36].

While bottleneck routing games are an important step in terms of integrating routing decisions with bottleneck objectives, they lack one fundamental tradeoff inherent in packet-switched communication networks: once a path is selected, a user increases the sending rate in case of low congestion and decreases it in case of high congestion. In this paper, we address this tradeoff by introducing bottleneck congestion games with *elastic* demands, where users can continuously vary their demands. Formally, there is a finite set of resources and a strategy of a player is a tuple consisting of a subset of resources and a demand. Resources have player-specific cost functions that are non-decreasing and strictly convex. Every user is associated with a non-decreasing strictly concave utility function measuring the received utility from sending at a certain demand rate (cf. [18, 31]). The goal of a user is to select both a subset of resources and a demand rate that maximizes the utility (from the demand rate) minus the congestion cost on the most expensive resource contained in the chosen resource set. Our model thus integrates as a special case (i) single-path routing (which is up to date standard as splitting packets over several routes leads

☆An extended abstract with parts of the results presented in this paper appeared in [14].

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[1] This research was carried out in the framework of Matheon supported by Einstein Foundation Berlin.