Weakly balanced contributions and solutions for cooperative games

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Weakly balanced contributions and solutions for cooperative games

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Abstract

We explore a relaxation of the balanced contributions property for solutions for TU games that requires the direction (sign) of one player's change of payoffs when another player leaves the game to equal the direction (sign) of the latter player's change of payoffs when the former leaves the game. There exists a large class of solutions that satisfy both efficiency and this weak balanced contributions property. The Shapley value is the unique solution that also obeys weak differential marginality.

Keywords: TU game, Shapley value, balanced contributions, weakly balanced contributions, weak differential marginality
2010 MSC: 91A12 *JEL:* C71, D60

1. Introduction

The Shapley value ([1]) probably is the most eminent one-point solution concept for cooperative games with transferable utility (TU games). Besides its original axiomatic foundation by Shapley himself, alternative foundations of different types have been suggested later on. Important direct axiomatic characterizations are due to [2] and [3]. [4] suggest an indirect characterization as the marginal contributions of a potential (function). [5] shows that the Shapley value can be understood as a von Neumann-Morgenstern utility. As a contribution to the Nash program, which aims at building bridges between cooperative and non-cooperative game theory, [6] implement the Shapley value as the outcome of the subgame perfect equilibria of a combined bidding and proposing mechanism, which is modeled by a non-cooperative extensive form game.

Among the direct characterizations, the one by [2] stands out by invoking only two properties, efficiency and the balanced contributions property. Efficiency says that the worth generated by the grand coalition is distributed without gains or losses among the players. The balanced contributions property requires that *the amount* one player gains or loses when another player leaves the game equals *the amount* the latter player gains or loses when the former player leaves the game.

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