Accepted Manuscript

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PII: S0167-6377(17)30180-3
DOI: https://doi.org/10.1016/j.orl.2017.10.009
Reference: OPERES 6287

To appear in: *Operations Research Letters*

Received date : 24 March 2017
Revised date : 12 October 2017
Accepted date : 12 October 2017

Funding, repo and credit inclusive valuation as modified option pricing

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Abstract

We take the holistic approach of computing an OTC claim value that incorporates credit and funding liquidity risks and their interplays, instead of forcing individual price adjustments: CVA, DVA, FVA, KVA. The resulting nonlinear mathematical problem features semilinear PDEs and FBSDEs. We show that for the benchmark vulnerable claim there is an analytical solution, and we express it in terms of the Black-Scholes formula with dividends. This allows for a detailed valuation analysis, stress testing and risk analysis via sensitivities.

Keywords: Funding costs, counterparty risk, credit risk, repo market, valuation adjustments, hedging
2010 MSC: 91G40, 60J28

1. Introduction

Prior to the financial crisis of 2007-2008, institutions tended to ignore the credit risk of highly-rated counterparties in valuing and hedging contingent claims traded over-the-counter (OTC), claims which are in fact bilateral contracts negotiated between two default-risky entities. Then, in just the short span of one month of 2008 (Sep 7 to Oct 8), eight mainstream financial institutions experienced critical credit events in a painful reminder of the default-riskiness of even large names (the eight were Fannie Mae, Freddie Mac, Lehman Brothers, Washington Mutual, Landsbanki, Glitnir and Kaupthing, to which we could also add Merrill Lynch).

One of the explosive manifestations of this crisis was the sudden divergence between the rate of overnight indexed swaps (OISs) and the LIBOR rate, pointing to the credit and liquidity risk existing in the interbank market. This forced dealers and financial institutions to reassess the valuation of OTC claims, leading to various adjustments to their book value.

It is difficult to do justice to the entire literature on such valuation adjustments, which intertwines two strands that have been developed in parallel by academics and practitioners. For a full introduction to valuation adjustments and all related references we refer to the first chapter of either Brigo et al. [15] or Crépey et al. [17].

All such adjustments may concern both over the counter (OTC) derivatives trades and derivatives trades done through central clearing houses (CCP), see for example Brigo and Pallavicini [16] for a comparison of the two cases where the full mathematical structure of the problem of valuation under possibly asymmetric initial and variation margins, funding costs, liquidation delay and credit gap risk is explored, resulting in BSDEs and semilinear PDEs. It is worth pointing out that the size of such derivatives markets remains quite relevant even post-crisis. At end of 2012, the market value of outstanding OTC derivative contracts was reported to be $24.7 trillion with $632.6 trillion in notional value (BIS 2013). Even if many deals are now collateralized in an attempt to avoid CVA altogether, contagion and gap risk may still result in important residual CVA, as was shown for the case of credit default swap trades in Brigo et al. [10].

As we mentioned above, the rigorous theory of valuation in presence of all such effects can be quite challenging, leading to models that are based on advanced mathematical tools such as semilinear PDEs or BSDEs, which make numerical analysis difficult and slow. See for example El Karoui et al. [18] for an example of how asymmetric interest rates, even in absence of credit risk, lead to BSDEs. The papers Brigo et al. [11] and Bichuch et al. [1] deal with the mathematical analysis of valuation equations in presence of all the abovementioned effects and risks, except KVA, for which we refer instead to Brigo et al. [12] for an indifference pricing approach. Biffis et al. [7] analyzes such effects in the area of life insurance contracts, and longevity swaps in particular.

Isolating and computing each individual adjustment is difficult because there is a marked interplay between them in pricing. Therefore, the causes of these adjustments are accounted for at the level of the contract payoffs and the resulting all-inclusive price is written as a solution to an advanced mathematical problem of the type mentioned above. Is there a case, even for a simple contract, where this all-inclusive price of an uncollateralized contract can be calculated analytically? We present here an answer in the affirmative.

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