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Characterizing rules in minimum cost spanning tree problems

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Abstract

A theorem of Bergantiños and Vidal-Puga (Characterization of monotonic rules in minimum cost spanning tree problems, International Journal of Game Theory 44 (2015) 835–868) states that in minimum cost spanning tree problems a rule satisfies separability and reductionism if and only if it is induced by an extra-costs function. In this article, we point out that the positivity of the extra-costs function implies the positivity of the rule; moreover, we prove that the rule further satisfies continuity if and only if the extra-costs function is continuous, and it satisfies symmetry if and only if the extra-costs function is symmetric.

Keywords: Cost sharing; minimum cost spanning tree; monotonicity; rule

1 Introduction

A minimum cost spanning tree problem is described as follows. A group of agents located at different geographical places want a particular service which can only be provided by a common supplier, called the source. Agents will be served through connections which entail some cost and they do not care whether they are linked directly or indirectly to the source. It is well known that the Borůvka-Kruskal algorithm [7, 8, 12] or the Jarník-Prim algorithm [10, 13] can be applied to produce a minimum cost spanning tree. Now the problem is how to divide the cost associated with this tree among the agents. In the literature there have been proposed many rules to solve this problem, such as the Bird rule [6], the Kar rule [11], and the Bergantiños-Vidal-Puga rule [4] etc. These rules satisfy many good properties, cf. [4]. Recently, Bergantiños and Vidal-Puga [5] characterized the set of rules satisfying reductionism and separability as the set of rules associated with extra-costs functions. So a good extra-costs function should induce a good rule. In this article, we point out that the positivity of the extra-costs function implies the positivity of the rule; moreover, we prove that the rule further satisfies continuity if and only if the extra-costs function is continuous, and it satisfies symmetry if and only if the extra-costs function is symmetric.

Our paper is organized as follows. In Section 2, we introduce the model of the minimum cost spanning tree problem and the rule characterization of Bergantiños and Vidal-Puga. In Section 3, we state our main results and the proofs.

2 The model

The minimum cost spanning tree problem is modeled as follows. Let $U = \{1, 2, \dots, |U|\}$ denote the finite set of possible agents, and let 0 be the source. A *spanning tree* on V_0, where $0 \in V_0 \subset U \cup \{0\}$, is an acyclic connected graph on V_0. Let $a = \{i, j\}$ be an edge linking i and j for $i, j \in V_0$ and its cost is $c_a = c_{ij}$. In this model, $c_{ii} = 0$ and $c_{ij} = c_{ji}$ for all $i, j \in V_0$. The matrix $C = (c_{ij})_{i,j \in V_0}$ is called the *cost* on V_0. The pair (V_0, C) is a *network*. Given a spanning tree T on V_0, the *cost* of T in (V_0, C) is defined as $c(T, C) = \sum_{a \in T} c_a$. A *minimum cost spanning tree* (*mcst* for brevity) in (V_0, C) is a spanning tree on V_0 with minimum cost. A mcst is not necessarily unique, but all mcst in (V_0, C) have the same cost, denoted by $m(V_0, C)$. We call such a network (V_0, C) a *minimum cost spanning tree problem* (*mcstp*

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