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Determination of in-duct sound power beyond the plane wave range using wall-mounted microphones

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A B S T R A C T

When studying the acoustic wave propagation in a duct, the frequency range can be divided into the low frequency plane wave range and the high frequency range with non-plane waves. In the low frequency range, the wave propagation is one-dimensional and the governing equations are rather simple. The larger the duct, the lower the frequency limit of the non-plane waves. Therefore, also taking into account the three-dimensional acoustic wave propagation is important, especially when considering the duct systems used in large machines. In practice often a harsh environment and immobile structures restrict the use of standardized noise measuring methods. For instance to characterize the exhaust noise of medium speed internal combustion engines (IC-engines) in situ, the in-duct sound pressures are measured using wall-mounted microphones. Then the low frequency range source sound power can be estimated by wave decomposition ("two-microphone method"). Often a three-microphone array is used to cover a sufficiently large frequency range. One way to formulate the sound pressure and sound power relationship in the high frequency range is to weight the sound pressures at the duct wall in one-third octave bands. The aim of this study is to extend the classical plane wave formulation by determining these weighting factors, so that a three-microphone array also can be used beyond the plane wave range. The results from numerical approach are compared to experimental data.

1. Introduction

Tighter legislation concerning environmental noise emission is challenging to machine manufacturers. The acoustic wave propagation in a channel, e.g. an exhaust duct, is of great importance when considering, for example, the noise from medium speed IC-engines, fans, or compressors. In general, noise from a source travels via the transmission path to the outside environment and finally to the receiver. When designing transmission path elements such as silencers or the after-treatment devices of an IC-engine exhaust system, the acoustic source data must be known.

The governing equations of the plane wave phenomena can be formulated and solved. In order to characterize the IC-engine as an acoustic source, it is possible to use multi-load methods [1], for example. Commercial codes designed for the analysis of one-dimensional, non-linear, time-varying compressible flows can be used in order to characterize an automotive IC-engine intake [2] or exhaust [3] as an acoustic source. Hynninen et al. [4] were the first to use these methods for a medium speed IC-engine exhaust system. Studying the plane-wave phenomena is enough if the channel is quite small, e.g. in the automotive intake or exhaust systems. In the larger ducts used in medium speed IC-engine exhaust systems, the frequency limit of the non-plane wave range is much lower. For example, at a temperature of 450 °C the first non-plane wave cut-on frequency in a typical automotive exhaust pipe of ø = 50 mm is 6319 Hz, whereas in the medium speed IC-engine with an exhaust duct of ø = 1600 mm this frequency is only 197 Hz. Thus, taking into account the three-dimensional effect in the acoustic wave propagation is important. Unfortunately, the high frequency phenomena are so complicated in practice that it is impossible to simulate them accurately. Therefore, there is a need to develop experimental methods so as to estimate the acoustic source characteristics, i.e. sound power, in the high frequency non-plane wave range. The low and high frequency range acoustic source data can be combined using a power-based formulation [5,6] and used as input data in the transmission path simulations with software developed for that purpose, e.g. a compact silencer system CSS [7].

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One measurable quantity in a duct is the sound pressure. The relationship of the sound pressure and sound power is not simple beyond the plane wave range. Bolleter and Crocker [8] and Bolleter et al. [9] studied several microphone arrangements and modal participation factors when estimating the modal spectra and sound generation from ducted fans or compressors. They concluded that the modal power spectra can be predicted with reasonable accuracy if the radial position of the microphones can be set arbitrarily. They also concluded that the flow noise can be suppressed using a long cylindrical windscreen, i.e. a Friedrich tube, when measuring the pressure fluctuations in the duct.

The sound pressure is also related to the source distribution. The sound power determination in hard-walled pipes was studied by Michalke [10,11]. Based on the theoretical results on the propagation of sound generated by various types of sources presented in [10], the aim in [11] was to study how the sound power spectrum, by means of pressure measurements, can be determined without assumptions about the nature of the sound field to be measured. As an alternative to the standard method [12], Michalke proposed sound power determination via the area-averaged cross-spectral density of sound pressure using microphones at N_r different radial positions. In acoustics, non-dimensional numbers are often used to generalize results. In duct acoustics the Helmholtz number is often used to compare the size of an object with the wavelength of a sound wave. Helmholtz number is commonly written as $He = kR$, where k is the wave number and R is the radius of the duct. Here the Helmholtz number will be used to generalize the results obtained from simulations to an arbitrary duct size and sound speed. Michalke concluded that the system cannot be solved satisfactorily at higher frequencies ($He > 8$) and the total number of measurements increases rapidly with the frequency. Later, Arnold [13] measured the area-averaged cross-spectra for a number N_ϕ of equidistant angular microphone positions. In this way he extended the useful frequency range up to Helmholtz number $He = 30$. The standardized in-duct method for determining the sound power radiated into a duct by fans and other air-moving devices [12] is based on the above-mentioned papers. Recently, Neise and Arnold [14] revised the standard and proposed new modal correction data in order to obtain the higher frequency sound power. Unfortunately, due to large immobile structures and a harsh environment, it is practically impossible to estimate the noise from the exhaust system of a medium speed IC-engine with the above-mentioned methods. Therefore, in case of medium speed IC-engine exhaust systems, the high frequency in-duct sound power determination is limited to the *in situ* measurements with wall-mounted microphones.

In order to estimate the in-duct acoustic power in the high frequency range Hynninen and Åbom studied and proposed experimental procedures [15]. They determined the high frequency in-duct sound power using wall-mounted microphones by extending the plane wave formulation with frequency band weighting factors. In their experiments, they used monopole, dipole and quadrupole excitations. Using the monopole type excitation correction factors was found to be a good approximation to estimate the source characteristics of a medium speed IC-engine [6]. In reality, the excitation of a typical duct acoustic problem is seldom known exactly. The source pressure distribution might also rotate and some specific acoustic modes can be excited particularly strongly.

The aim of this study is to generalize the method by determining the new 1/3 octave frequency band sound power weighting factors via simulations for multi-modal excitation, i.e., excitating all possible in-duct modes. These simulated weighting factors are compared to the experimentally determined weighting factors for monopole excitation [15]. Also the non-plane wave frequency range in-duct sound power formulation presented by Hynninen and Åbom [6] is clarified in detail. Another aim is to find the best three-microphone configuration which can be used with wall-mounted microphones not only in the plane wave range but also to estimate the high frequency non-plane wave range acoustic power. The suggested method extends the low frequency in-duct source characterization also to the high frequency range with non-plane waves. This is of great importance when characterizing IC-engine exhaust systems *in situ* with wall-mounted microphones, for example.

2. Theory

The acoustic wave propagation in a fluid can be described by the time harmonic inhomogeneous Helmholtz wave equation

$$\nabla \cdot \left(-\frac{1}{\rho}(\nabla p - \mathbf{q}) \right) - \frac{\omega^2 p}{\rho c^2} = \mathbf{Q}, \tag{1}$$

where p is the acoustic pressure, ρ is the density, c is the speed of sound, ω is the angular frequency, $\mathbf{q}$ describes the dipole source and $\mathbf{Q}$ the monopole source. By solving Eq. (1) with different sources the frequency response can be determined. Here it will be assumed that the density is constant and that the field is excited by a monopole.

In a duct, only a finite set of shapes for transversal pressure fields can propagate. The solution to the wave equation in a uniform circular straight duct can be written as a superposition of modal pressures as

$$\hat{p}(x, \mathbf{r}) = \sum_{(m,n)} \hat{p}_{mn}(x, \mathbf{r}), \tag{2}$$

where x is the coordinate along the duct axis, $\mathbf{r}$ is the position vector over the duct cross-section and m, n are the number of radial and circumferential nodal lines of the modal pressure field $\hat{p}_{mn}$, respectively. According to Munjal [16] for example, the modal pressures can be described using the downstream and upstream propagating waves as

$$\hat{p}_{mn}(x, \mathbf{r}) = \hat{p}_{mn+}\psi_{mn+}(\mathbf{r})e^{-jk_{xmn+}x} + \hat{p}_{mn-}\psi_{mn-}(\mathbf{r})e^{jk_{xmn-}x}, \tag{3}$$

where $\hat{p}_+$ and $\hat{p}_-$ are the complex valued acoustic pressures propagating downstream (+) and upstream (−), ψ_+ and ψ_- are the orthonormal eigenfunctions which are dependent on the cross-section shape and the flow profile and k_+ and k_- are the wave numbers. For uniform flow, the wave numbers are defined by

$$k_{xmn\pm} = \frac{\mp M k_0 + \sqrt{k_0^2 - (1 - M^2)k_{rmn}^2}}{1 - M^2}, \tag{4}$$

where M is the Mach number, k_0 is the ordinary wave number, and k_{rmn} are the well-known transversal wave numbers defining the cut-on frequencies. The nodal lines of some non-plane wave modes in a circular duct and corresponding Helmholtz cut-on frequencies $He = k_{rmn}R$ are presented in Table 1. The corresponding speeds of the downstream and upstream propagating wave modes are

$$c_{mn\pm} = \frac{\omega}{k_{xmn\pm}}. \tag{5}$$

Writing the acoustic velocity field as a superposition of the modal axial velocities as

$$\hat{u}(x, \mathbf{r}) = \sum_{(m,n)} \hat{u}_{mn}(x, \mathbf{r}), \tag{6}$$

and use the linearized equation of motion [16] assuming harmonic time dependence, the modal axial velocities can be written as

$$\hat{u}_{mn}(x, \mathbf{r}) = \frac{\hat{p}_{mn+}}{\rho c_{mn+}}\psi_{mn+}(\mathbf{r})e^{-jk_{xmn+}x} - \frac{\hat{p}_{mn-}}{\rho c_{mn-}}\psi_{mn-}(\mathbf{r})e^{jk_{xmn-}x}. \tag{7}$$