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# A new random interval method for response analysis of structural–acoustic system with interval random variables

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For the response analysis of the structural–acoustic system without sufficient information, an interval random model is introduced. In the interval random model, the uncertain parameters are assumed as random variables, while some probability distribution parameters of random variables are modeled as interval parameters instead of precise values. Based on the interval random model, the interval random dynamic equation is constructed and a new random interval method named as the random moment – change of variable interval stochastic perturbation method (RM-CVISPM) is proposed. In RM-CVISPM, the response of the structural–acoustic system is approximated as a linear function of interval random variables based on the first-order stochastic perturbation analysis. Then, the expectation and variance of the obtained response are calculated by the random moment method. The probability density of the obtained response is yielded by the change of variable technique. Finally, the intervals of expectation, variance and probability density of response are estimated by an interval perturbation method. Numerical results on a shell structural–acoustic model and an automobile passenger compartment with flexible front panel verify the effectiveness and efficiency of RM-CVISPM.



## 1. Introduction

The structural–acoustic system, which consists of the vibrating structure, the acoustic cavity and the coupled interface, is widely existed in the automobile, train, airplane and other transports. The response analysis of the structural–acoustic system plays an important role in the noise and vibration control of these transports. Traditional numerical methods for the response analysis of the structural–acoustic system are based on deterministic parameters [1]. However, due to the effects of manufacturing/assembling errors, aggressive environment factors and unpredictable external excitations, the uncertainties associated with the structural–acoustic system are unavoidable. The response of the structural–acoustic system is very sensitive to these uncertainties [1]. Without considering these uncertainties, the response of the structural–acoustic system obtained by deterministic numerical methods may be unreliable.

Up to now, the random model is still considered as the most valuable mathematical model to treat with the uncertainties existing in engineering practices. In the random model, the uncertain parameters are assumed as random variables whose probability distributions are defined unambiguously. For the uncertain system with random variables, a lot of probabilistic methods, such as the Monte Carlo simulation method [2,3], the spectral stochastic method [4–6] and the stochastic perturbation method [7–10], have been developed. Based on these probabilistic methods, the expectation, variance and even the probability density of the response of the stochastic system with random variables can be obtained. Unfortunately, in the early stage of design, the objective information to determine the probability distributions of uncertain parameters is limited. In these cases, some suitable assumptions on the probability distributions of uncertain parameters have to be made. However, these assumed probability distributions may be unreliable and the results obtained by probabilistic methods based on the assumed probability distributions may be incorrect [11].

To overcome the above deficiency of random model, an interval model is developed. In the interval model, the uncertain parameters with limited sampled data are treated as the interval variables whose variational ranges are well-defined but the information on the probability distributions within the variational ranges is missing. For the response analysis of the uncertain system with interval variables, a lot of interval methods, such as the Gaussian elimination method [12,13], the vertex method [14,15] and the interval perturbation method [16–19] have been proposed. There are two inherent drawbacks existing in the interval methods. The first

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one is that only the information on the variational range of response can be obtained by the interval method. Compared with the variational range, the probability distribution of response can provide more important information (such as the expectation, variance and even the probability density of response) for the design and optimization of the uncertain structural–acoustic system. The second one is that the results yielded by interval methods may be ultra-conservative due to the effect of the dependency phenomenon which appears when the interval parameters occur more than once in an interval expression [20].

To better represent the uncertainties of parameters without sufficient information, a hybrid uncertain model has been proposed by Elishakoff and Colombi [21,22]. In the hybrid uncertain model, the uncertain parameters are assumed as random variables, while some probability distribution parameters of random variables with limited sampled data are modeled as interval parameters. This hybrid uncertain model, called as the interval random model in this paper, is an extension of the random model. Compared with the random model, the main advantage of the interval random model is that only the variational ranges, but not the precise values of the probability distribution parameters have to be obtained. Compared with the interval model, the main advantage of the interval random model is that the probability distribution characteristics of uncertain parameters which are completely neglected by the interval model can be considered.

Recently, some great successes have been achieved in the reliability analysis of structures with interval random variables [23–27]. However, the research on the response analysis of the complex engineering system with interval random variables is still in its preliminary stage. Xia et al. have proposed a hybrid random interval method (HRIM), which is based on the combination of the matrix perturbation method, the vertex method and the random interval moment method, to predict the intervals of expectation and variance of response of the acoustic field with interval random variables [28]. HRIM can be extended to the response analysis of the structural–acoustic system with interval random variables. However, only the intervals of expectation and variance of response can be yielded by HRIM. The variational range of probability density which plays an important role in the design and optimization of uncertain structures cannot be obtained. In the probability theory, the probability density function describes the relative likelihood of a random variable at a given value. Thus, if the interval of the probability density of response is yielded, the variational range of the relative likelihood of response at any considered value can be obtained directly. In engineering practices, the variational range of the relative likelihood of response at the considered value can be of guidance for design and optimization of the structural–acoustic system with interval random variables. Therefore, it is desired to develop a new hybrid interval random method which can be used to predict not only the intervals of expectation and variance but also the interval of probability density of response.

In this paper, a new hybrid interval random method named as the random moment – change of variable interval stochastic perturbation method (RM-CVISPM) is proposed. The main procedure of RM-CVISPM for the response analysis of the structural–acoustic system with interval random variables is divided into two steps. The first one is the random response analysis of the structural–acoustic system with interval random variables. In this step, the response of the structural–acoustic system with interval random variables is approximated as a linear function of interval random variables based on the first-order stochastic perturbation analysis. The expectation and variance of the obtained response are calculated by the random moment method [29]. The probability density of response is yielded by the change of variable technique based on invertible relationships between the response and interval random variables [30]. The second step is the interval response analysis of the structural–acoustic system with interval random variables. The obtained expectation, variance and probability density of response are the functions of probability distribution parameters expressed as interval variables. Thus, the uncertainties of expectation, variance and probability density of response rising from interval variables can be expressed as the interval functions whose variational ranges can be calculated by an interval perturbation method. In the interval perturbation method, the expectation, variance and probability density of response are expanded to the first-order Taylor series at the mean values of interval parameters. For various complex engineering problems, it may be difficult or even impossible to obtain the analytical solutions of the first-order derivatives of expectation, variance and probability density. Thus, their first-order derivatives are approximated by the central difference method which can be easily implemented in engineering practices. And then the intervals of expectation, variance and probability density of response are calculated by the interval operation.

The remainder of this paper is organized as follows. In Section 2, the dynamic equation of the structural–acoustic system with interval random variables is derived. In Section 3, the random response analysis of the structural–acoustic system with interval random variables is presented. In Section 4, the interval response analysis of the structural–acoustic system with interval random variables is presented. In Section 5, two numerical examples including a shell structural–acoustic model with interval random variables and an automobile passenger compartment with interval random variables are provided to investigate the efficiency and effectiveness of RM-CVISPM. Some conclusions are given in Section 6.

## 2. The dynamical equation of the structural–acoustic system with interval random variables

For a thin-walled structure, the interaction between the vibrating structure and the acoustic cavity cannot be neglected. The vibrating structure, the acoustic cavity and the coupled interface between the vibrating structure and the acoustic cavity make up a structural–acoustic system. Without considering the structural damping, the dynamical equation of the structural–acoustic system under the time harmonic external excitation can be expressed as

$$\begin{bmatrix} \mathbf{K}_s - \omega^2\mathbf{M}_s & -\mathbf{H} \\ \rho_f\omega^2\mathbf{H}^T & \mathbf{K}_f - \omega^2\mathbf{M}_f \end{bmatrix} \begin{Bmatrix} \mathbf{u}_s \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_s \\ \mathbf{F}_f \end{Bmatrix} \tag{1}$$

where $\omega$ is the angular frequency of external excitation; $\rho_f$ is the density of fluid in the acoustic cavity; $\mathbf{K}_s$ and $\mathbf{M}_s$ are the stiffness matrix and the mass matrix of the vibrating structure; $\mathbf{K}_f$ and $\mathbf{M}_f$ are the stiffness matrix and the mass matrix of the acoustic cavity; $\mathbf{H}$ is the spatial coupled matrix; $\mathbf{F}_s$ and $\mathbf{F}_f$ are the generalized force vectors loading on the vibrating structure and the acoustic cavity, respectively; $\mathbf{u}_s$ and $\mathbf{p}$ are the displacement vector of the vibrating structure and the sound pressure vector in the acoustic cavity, respectively. The detailed derivation of Eq. (1) can be referred to Ref. [31].

In order to simplify the process of analyzing the dynamical equation of the structural–acoustic system with interval random variables, we rewrite Eq. (1) as the following form

$$\mathbf{ZU} = \mathbf{F} \tag{2}$$

where $\mathbf{Z}$ is the dynamic stiffness matrix of the structural–acoustic system; $\mathbf{U}$ is the response vector; $\mathbf{F}$ is the force vector. They can be expressed as

$$\mathbf{Z} = \begin{bmatrix} \mathbf{K}_s - \omega^2\mathbf{M}_s & -\mathbf{H} \\ \rho_f\omega^2\mathbf{H}^T & \mathbf{K}_f - \omega^2\mathbf{M}_f \end{bmatrix}, \quad \mathbf{U} = \begin{Bmatrix} \mathbf{u}_s \\ \mathbf{p} \end{Bmatrix}, \quad \mathbf{F} = \begin{Bmatrix} \mathbf{F}_s \\ \mathbf{F}_f \end{Bmatrix} \tag{3}$$