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Coupling mechanism analysis of structural modes and sound radiations of a tire tread band based on the S-mode technique

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ABSTRACT

Structural vibration is an important noise source for road tires. However, the generation mechanism of tire noise from vibration has not been so well understood because of the complicated coupling relations between vibrating modes and sound power radiations. In the present paper, sound power radiation of a vibrating tire tread band is investigated by a newly developed structure-dependent radiation mode (s-mode) technique. Differing from conventional acoustic radiation modes (a-modes) which are defined as a set of velocity distributions, s-modes are defined as a set of modal velocity distributions. Although both definitions allow the sound power to be described as a set of independently radiation terms, the s-mode description can set a direct relation between structural modes and s-modes for vibrating structures. Consequently, the s-mode technique can be very helpful on analyzing the coupling effects between the structural modes of a vibrating tire and its sound radiation. In the first instance, the tire tread band is modelled as a thin, cylindrical shell with both ends simply supported. In this case, complete analytical solutions are available for both structural modes and radiation modes of the model. Numerical investigations have shown that (1) each s-mode and the same ordered a-mode are associated with the same vibrating modes; (2) only low ordered radiation modes (either s-modes or a-modes) can radiate sound power efficiently into the surroundings within the frequency range of interest; (3) below the ring frequency, the dominant s-modes tend to be associated not only with the resonant modes, but also with the non-resonant modes whose nature frequencies are below the frequency of interest; (4) after the ring frequency, however, the sound power tends to be dominated by the vibrating modes which have natural frequencies nearby the frequency of interest, and at the same time, are associated with low axial wavenumbers.

1. Introduction

As a major contributor to the noise levels of passenger vehicles, tire vibration and the associated noise radiation into the surroundings have been of great interest for both academica and industry [1–4]. These problems are mostly investigated by combining conventional dynamic and acoustic modelling techniques, e.g. wave- and mode-based analytical methods [5–8], finite-element-analysis and/or boundary-element-analysis based numerical methods [9–12], as well as experimental approaches [13–15], etc. However, the coupling relations between structural modes and sound radiation of a vibrating tire, so far, have been found not so clear [15] mainly because of the complicated radiation mechanism of tire vibration modes [1,3]. It is desirable to have a deeper understanding on how structural modes can affect the sound power radiation of a vibrating tire.

Conventional acoustic radiation modes (a-modes) have been a useful tool on investigating the coupling relations between structural modes and the sound radiation of vibrating structures [16,17]. The a-mode approach has also been used to investigate the tire's structural wave propagation and sound radiation, e.g. in Refs. [13–15]. However, since a-modes are independent of the structure's vibration: i.e. they depend only on the geometry of the problem and frequency, the existing techniques still show some difficulties in obtaining the quantitative insight on explaining, for example, why most of a tire's structural vibration does not contribute to the radiation [14,15].

Recently, a concept of structure-dependent radiation modes (s-modes) has developed by extending the definition of a-modes [18]. Different from a-modes which are defined as a set of velocity distributions along the surface of a vibrating structure, s-modes are

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defined as a set of modal velocity distributions of the structure [18]. Although the two sets of radiation modes possess similar acoustic properties, s-modes are found to be with better modal radiation efficiencies (and hence better convergence) than a-modes [18]. Particularly, the s-mode description can help to set a direct relation between the independent vibrating terms (structural modes) and independent radiation terms (s-modes) of a vibrating structure. The inherent coupling mechanism between structural modes and sound radiation properties of the structure can then be determined in an explicit way.

Since the acoustic properties of s-modes are found somehow depending on structure types, i.e., the coupling mechanism between structural modes and sound radiation observed from beam- and plate-like structures may be different from that from cylinder-like structures, the inherent coupling relations between tire's structural modes and its sound radiation are particularly investigated in the present study based on the s-mode approach. It is known that tire structures can usually be approximated by equivalent thin cylindrical shells under arbitrary loads [8–11]. In the present study, therefore, a circular cylindrical shell model under a harmonic force excitation acting at the center line of the tread band, same as that employed in Ref. [10], is set up. Meanwhile, the tread band model is assumed to be of both ends simply supported. In this case, both the structural modes and radiation modes of the tread band can be obtained by analytical expressions [19,20].

The paper is organized as following. First, the definition of structure-dependent radiation modes (s-modes) is reviewed in Section 2, together with a discussion on the coupling relations between the modal response of a structural mode and the associated sound radiation via different orders of s-modes. Analytical solutions are then provided in Section 3 to predict both the vibration and acoustic radiation for a simplified tread band model. Numerical investigations are made in Section 4 in which the sound radiations from s-modes and a-modes of the tread band are compared and the coupling relations between the radial modes and the dominant s-modes are then discussed. Main results are finally concluded in Section 5.

2. Structure-dependent radiation modes and the coupling with vibrating modes

In this section, the definition of structure-dependent radiation modes (s-modes) is first re-capitulated in Section 2.1 for the sake of convenience of reader. The principle for analyzing the coupling mechanism between the sound power radiation from the s-modes of a vibrating structure and its structural modes are then described in Section 2.2.

2.1. Structure-dependent radiation modes (s-modes)

Assume the whole vibrating surface of the structure can be divided into an array of equal sized elemental radiators, each with an area of S, and is small enough to be treated as vibrating entirely in-phase and radiating sound power into free-space. The total acoustic power output from the vibrating surface of the structure can then be formulated as [16,17]

$$W = \frac{S}{2}\mathrm{Re}\{\mathbf{v}^H\mathbf{Z}\mathbf{v}\} = \mathbf{v}^H\mathbf{R}\mathbf{v} \tag{1}$$

where $\mathbf{v}$ is the surface velocity vector associated with this array of elemental radiators, $\mathbf{Z}$ is the associated acoustic impedance matrix, and $\mathbf{R} = \frac{S}{2}\mathrm{Re}\{\mathbf{Z}\}$. Clearly, $\mathbf{R}$ is real, symmetric and positive definite, and proportional to the radiation resistant matrix [17].

By modal approach, $\mathbf{v}$ can be expressed in terms of structural modes as

$$\mathbf{v} = \boldsymbol{\Phi}\mathbf{a} \tag{2}$$

where $\boldsymbol{\Phi}$ is a structural mode matrix with each column corresponding a mode shape of the structure, and $\mathbf{a}$ is the associated modal coefficient vector. Clearly, the sizes of $\boldsymbol{\Phi}$ are determined by both the number of elemental radiators, I, and the number of structural modes, N, involved in the vibration.

Substituting Eq. (2) into Eq. (1), gives

$$W = \mathbf{a}^H\boldsymbol{\Phi}^H\mathbf{R}\boldsymbol{\Phi}\mathbf{a} \tag{3}$$

Define

$$\mathbf{M} = \boldsymbol{\Phi}^H\mathbf{R}\boldsymbol{\Phi} \tag{4}$$

Obviously, $\mathbf{M}$ is Hermitian. Eigen-decomposition of $\mathbf{M}$ then gives

$$\mathbf{M} = \mathbf{P}^H\boldsymbol{\Sigma}\mathbf{P} \tag{5}$$

where $\mathbf{P}$ is a orthogonal matrix composed by N rows of eigenvectors, and $\boldsymbol{\Sigma}$ is a diagonal matrix composed of the N eigenvalues. Because the radiated power is always positive for any non-zero choice of $\mathbf{a}$, $\mathbf{M}$ is positive definite, and hence $\boldsymbol{\Sigma}$ is composed by real and positive eigenvalues with $\sigma_1 > \sigma_2 > \ldots > \sigma_n$.

From Eq. (5), Eq. (3) can then be re-written as

$$W = \mathbf{a}^H\mathbf{P}^H\boldsymbol{\Sigma}\mathbf{P}\mathbf{a} = \mathbf{z}^H\boldsymbol{\Sigma}\mathbf{z} = \sum_{n=1}^{N}|z_n|^2\sigma_n \tag{6}$$

Eq. (6) thus shows that the radiated sound power can be described as a set of independent radiation modes, which are closely affected by the dynamic properties of the structure.

To distinguish from the classical acoustic radiation modes (a-modes) defined in [17], the radiation terms in Eq. (6) are therefore defined as structure-dependent radiation modes (s-modes) [18], whose mode shapes and associated radiation efficiencies are determined by rows of $\mathbf{P}$ and the diagonal elements of $\boldsymbol{\Sigma}$, respectively.

Because the number of s-modes generated is generally much less than that of a-modes for large scaled structures and/or high frequency vibrations, s-modes are found to possess better convergence than the a-modes [18].

Meanwhile, since Eq. (6) can set a direct link between the independent vibrating terms (structural modes) and independent radiation terms (s-modes) of a vibrating structure, the s-mode description thus provides a very useful tool on investigating the coupling relations between vibrating modes and the associated sound power [18].

2.2. Coupling relations between a single structural mode and the s-modes

If it is assumed that only a single natural mode, say, the nth vibrating mode, is excited, the radiated sound power, by Eqs. (5) and (6), becomes

$$W = |a_n|^2M_{nn} = |a_n|^2\sum_{n'=1}^{N}|p_{n'n}|^2\sigma_{n'} \tag{7}$$

Eq. (7) shows that, even if a single structural mode is excited, the radiated acoustic power tends to be associated with many s-modes. Since lower ordered s-modes tend to have relatively much higher radiation efficiencies than the higher ordered ones at low frequencies while their radiation efficiencies tend to be similar as frequency goes up [17], one can thus deduce from Eq. (7) that, at low frequencies, the vibrating mode tends to radiate power mainly