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Transmission of a spherical sound wave through a single-leaf wall: Mass law for spherical wave incidence

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ABSTRACT

This paper examines the sound insulation of a single-leaf wall driven by a spherical wave. The transmitted sound field of an infinite elastic plate under a spherical wave incidence is theoretically analyzed and insulation mechanisms are considered. The displacement of the plate is formulated using the Hankel transform in wavenumber space and the transmitted sound pressure in the far-field is obtained by Rayleigh's formula in an explicit closed form. Moreover, a reduction index is also derived in a closed form by introducing an approximation into the vibration characteristics of the plate. Deterioration of the insulation performance under the spherical wave incidence is caused by an apparent decrease of wall impedance that depends on the directivity of the transmitted sound wave. The mass law for a spherical wave incidence is different from that for a normal plane wave incidence: doubling the weight of the wall or the frequency gives an increase of 3 dB (c.f. 6 dB for a normal plane wave incidence), which is also smaller than the field incidence mass law.

1. Introduction

The fundamental theory of the mass law for the sound insulation of a single-leaf wall is widely used in architectural acoustics today [1]. The mass law is simply derived from the relationship between a normal plane wave incidence and the mechanical impedance of an infinite plate, and indicates that doubling the weight of the wall or the frequency yields a 6 dB increase in the reduction index.

The normal incidence mass law has been conventionally developed into random incidence conditions [2] by considering oblique incidence, because normal incidence is not practical for evaluating actual walls. Random incidence is obtained by averaging the transmission coefficients for oblique incidences over a hemisphere. This approach, which is based on a completely diffuse sound field, does not fully reflect the actual sound field incidence conditions in rooms, and so various methods of truncating the angle of incidence up to a certain limit angle have been proposed [3,4].

The angle of incidence also affects the bending vibration of the plate, which is well known as the coincidence effect. When elastic plates are accompanied by bending vibrations due to an oblique plane wave incidence, the reduction index becomes lower than the mass law above a critical frequency. A fundamental theory for the range above the critical frequency has been established [5], and a number of researchers have developed it further [6,7].

In order to increase the insulation performance of walls, various multiple-leaf walls have been proposed and widely used in actual buildings. For example, the insulation characteristics of a double-leaf wall, each of which is an isolated single-leaf wall, are basically governed by the sum of the mass law of each leaf. In practice, sound is transmitted via the structural coupling and the acoustic coupling due to the air between the two leaves and so the insulation performance is lower than that expected by a simple sum of the mass law [8]. Since the coupling mechanisms between leaves are very complicated, a number of theoretical and empirical models for predicting the sound insulation have been proposed [9,10].

In this way, the mass law of a single-leaf wall is the fundamental principle of sound insulation in architectural acoustics, which consider various plane wave incidences, i.e. normal, oblique and diffuse (random). In actual buildings, however, a wall may be excited by a small sound source nearby, in which case, the wall is expected to be driven not by a plane wave but by a spherical wave. Takahashi et al. [11] initially analyzed the sound insulation for a spherical wave incidence. In the literature, they gave a numerical solution for executing the wavenumber space integral for the transmitted sound power. Based on numerical examples, they concluded that the reduction index for the spherical wave incidence was lower than that for the plane wave incidence. Villot et al. [12] also studied the transmission of sound through a wall excited by a small sound source by using a numerical solution, and obtained similar results.

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As described above, the sound insulation for spherical wave incidence has been investigated to some extent, but neither the reason why the reduction index for a spherical wave is lower than that for a plane wave, nor the quantitative difference between them is clear. To gain a physical insight into the differing insulation performance between a spherical wave incidence and normal plane wave incidence, this paper theoretically analyzes the transmitted sound field of an infinite elastic plate driven by a spherical wave, and obtains a solution in an explicit closed form. Using the solution, the insulation mechanism under the spherical wave incidence is clarified. Furthermore, a mass law formula for a spherical wave incidence is derived and discussed in comparison with that for normal and diffuse plane waves.

2. Theory

Consider an infinite elastic plate lying in the plane, $z = 0$, in Fig. 1, that vibrates under spherical wave incidence from a point source, $(0, 0, d_s)$. The sound pressure, $p_1(\mathbf{r})$, at a certain point, $\mathbf{r}$, in region I is expressed by the following integral:

$$p_1(\mathbf{r}) = p_0(\mathbf{r}) + p_0'(\mathbf{r}) + \iint_{S_1} \frac{\partial p_1(\mathbf{r}_0)}{\partial n_0} G(\mathbf{r}|\mathbf{r}_0) dS_0, \tag{1}$$

where $p_0(\mathbf{r})$ is the direct sound pressure from a point source and $p_0'(\mathbf{r})$ is the pressure contributed from its image source. The double integral denotes the integral over all regions of the boundary, i.e., the plate's source side surface, S_1 and n_0 denotes the outward normal of the region I. $G(\mathbf{r}|\mathbf{r}_0)$ denotes Green's function satisfying the Neumann condition for a single boundary with infinite extent, and is as follows:

$$G(\mathbf{r}|\mathbf{r}_0) = \frac{\exp(ik_0|\mathbf{r}-\mathbf{r}_0|)}{4\pi|\mathbf{r}-\mathbf{r}_0|} + \frac{\exp(ik_0|\mathbf{r}-\mathbf{r}_0'|)}{4\pi|\mathbf{r}-\mathbf{r}_0'|}, \tag{2}$$

where k_0 is the acoustic wavenumber in air with ω the angular frequency and c_0 the sound speed in air. The time dependence of $\exp(-i\omega t)$ is suppressed throughout. In region II, the sound pressure at a certain point, $\mathbf{r}$, $p_2(\mathbf{r})$ is written in a similar form:

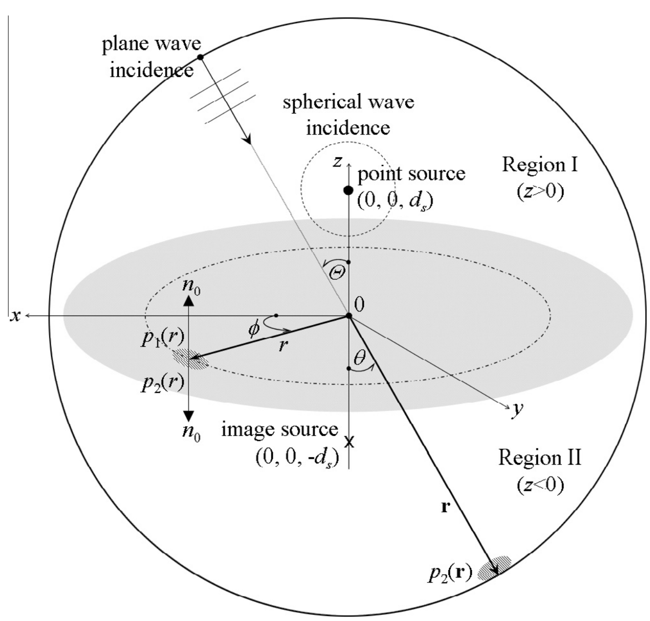

Fig. 1. Analytical model in cylindrical coordinates. An infinite elastic plate (shaded) lies in the plane, $z = 0$. The plate vibrates under spherical wave incidence from a point source, $(0, 0, d_s)$.

$$p_2(\mathbf{r}) = \iint_{S_2} \frac{\partial p_2(\mathbf{r}_2)}{\partial n_0} G(\mathbf{r}|\mathbf{r}_0) dS_0, \tag{3}$$

where the double integral in this equation denotes the integral over all regions of the boundary, i.e., the plate's back side surface, S_2, and n_0 denotes the outward normal in region II. The boundary conditions on the plate surfaces are:

$$\left.\frac{\partial p_1(\mathbf{r})}{\partial \mathbf{n}_0}\right|_{z=0} = -\left.\frac{\partial p_1(\mathbf{r})}{\partial z_0}\right|_{z=0} = -\rho_0\omega^2 w(\mathbf{r}) : \text{source side}, \tag{4}$$

$$\left.\frac{\partial p_2(\mathbf{r})}{\partial \mathbf{n}_0}\right|_{z=0} = \left.\frac{\partial p_2(\mathbf{r})}{\partial z_0}\right|_{z=0} = \rho_0\omega^2 w(\mathbf{r}) : \text{back side}, \tag{5}$$

where $w(\mathbf{r})$ is the displacement of the plate and ρ_0 is the air density. Considering these conditions and using the cylindrical coordinate system, $\mathbf{r} = (\rho, z) = (r, \phi, z)$ and $\mathbf{r}_0 = (\rho_0, z) = (r_0, \phi_0, z_0)$, $p_1(\mathbf{r})$ and $p_2(\mathbf{r})$ become:

$$p_1(\mathbf{r}) = p_0(r,0) + p_0'(r,0) - \rho_0\omega^2 \int_0^\infty W(k) \times \frac{\exp\left(-\sqrt{k^2-k_0^2}|z|\right)}{\sqrt{k^2-k_0^2}} J_0(kr)kdk, \tag{6}$$

$$p_2(\mathbf{r}) = \rho_0\omega^2 \int_0^\infty W(k) \frac{\exp\left(-\sqrt{k^2-k_0^2}|z|\right)}{\sqrt{k^2-k_0^2}} J_0(kr)kdk, \tag{7}$$

where the polar angle, ϕ is suppressed because the problem is axisymmetrical. J_0 denotes the Bessel function of order zero and k is the transform variable. $P_j(k)$ and $W(k)$ are the angular spectrums with respect to r of $p_j(r)$ and $w(r)$, respectively, as defined by the following Hankel transform pairs:

$$\begin{cases} P_j(k,z) = \int_0^\infty p_j(r,z) J_0(kr) rdr \quad j = 1,2 \\ p_j(r,z) = \int_0^\infty P_j(k,z) J_0(kr) kdk \quad j = 1,2 \end{cases}, \tag{8}$$

$$\begin{cases} W(k) = \int_0^\infty w(r) J_0(kr) rdr \\ w(k) = \int_0^\infty W(r) J_0(kr) kdk \end{cases}. \tag{9}$$

Since the pressure of spherical waves from the real and image source become identical on the plate's surface, i.e., $p_0(r) = p_0'(r)$ on $z = 0$, these can be written by considering the amplitude of the spherical wave from a unit power point source, i.e., $(8\pi\rho_0 c_0)^{1/2}$:

$$p_0(r,0) = p_0'(r,0) = \frac{\sqrt{8\pi\rho_0 c_0}\exp\left(-ik_0\sqrt{r^2+d_s^2}\right)}{4\pi\sqrt{r^2+d_s^2}}. \tag{10}$$

The plate is forced into vibration by the difference in sound pressure between both sides of the plate. In accordance with the classical thin plate theory, the equation of motion for the plate is as follows:

$$(D\nabla^4 - \rho_p h\omega^2) w(r) = p_1(r,0) - p_2(r,0), \tag{11}$$

where $D = E(1 - i\eta)h^3/12(1 - \mu^2)$ is the flexural rigidity of the plate with E Young's modulus, h the thickness, η the loss factor μ the Poisson's ratio and ρ_p the density of the plate. Solving Eqs. (6), (7), (10), and (11) for $W(k)$ by using the Hankel transform gives the displacement of the plate in the wavenumber space [13]:

$$W(k) = \frac{2\sqrt{8\pi\rho_0 c_0}\exp\left[-d_s\sqrt{k^2-k_0^2}\right]}{4\pi\sqrt{k^2-k_0^2}} \left[2\frac{\rho_0\omega^2}{\sqrt{k^2-k_0^2}} - (Dk^4 - \rho_p h\omega^2)\right]^{-1}. \tag{12}$$