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Optimal Berry–Esseen bound for parameter estimation of SPDE with small noise[☆]

Yoon Tae Kim, Hyun Suk Park [*]

Department of Statistics and Institute of Statistics, Hallym University, Chuncheon, Gangwon-Do 200-702, South Korea

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ABSTRACT

We investigate a rate of convergence on asymptotic normality of the maximum likelihood estimator (MLE) for parameter θ appearing in parabolic SPDEs of the form

$$du^{\epsilon}(t,x) = (A_0 + \theta A_1)u^{\epsilon}(t,x)dt + \epsilon dW(t,x),$$

where A_0 and A_1 are partial differential operators, W is a cylindrical Brownian motion (CBM) and $\epsilon \downarrow 0$. We find an optimal Berry–Esseen bound for central limit theorem (CLT) of the MLE. It is proved by developing techniques based on combining Malliavin calculus and Stein's method.

1. Introduction

Recently the study of statistical inference for the parameters, involved in infinite dimensional diffusions or SPDE, has been much studied (see, e.g., Bishwal, 2008 and Prakasa Rao, 2001, and the references therein). Authors of Hübner and Rozovskii (1995) investigate asymptotic properties of the MLEs for parameter θ occurring in parabolic SPDE's of the form

$$du(t,x) = (A_0 + \theta A_1)u(t,x) + dW(t,x), \quad t \le T, \; x \in G, \tag{1}$$

where A_0 and A_1 are partial differential operators of orders m_0 and m_1, and W is a cylindrical Brownian motion (CBM) in $L_2(G)$, G is a bounded domain in $\mathbb{R}^d$. They estimate parameter θ in Eq. (1) with appropriate initial and boundary conditions.

In reality, the available data for parameter estimation is given as only a finite-dimensional projection $u^N = (u_1(t), \ldots, u_N(t))$ of the solution of Eq. (1) corresponding to a finite number of Fourier coefficients of $u(t,x)$ in x-variable. In Hübner and Rozovskii (1995), authors use the number N of Fourier coefficients to obtain the MLE $\hat{\theta}_N$ and investigate the asymptotic properties of $\hat{\theta}_N$ as $N \uparrow \infty$. They study the usual range of asymptotic problems in parameter estimation such as consistency, asymptotic normality and asymptotic efficiency.

In particular, authors in Hübner and Rozovskii (1995) prove the problem of asymptotic normality of the MLE $\hat{\theta}_N$ as $N \uparrow \infty$.

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* Corresponding author.
E-mail addresses: ytkim@hallym.ac.kr (Y.T. Kim), hspark@hallym.ac.kr (H.S. Park).

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Theorem 1 (*Huebner and Rozovskii*). *Assume that $m_1 \geq m-(d/2)$, where $2m = \max\{m_0, m_1\}$ and set*

$$\mathbb{I}_N = \begin{cases} \frac{\zeta}{4\tau} TN^{2\tau} + o(N^{2\tau}) & \text{if } m_1 > m - \frac{d}{2}, \\ \frac{\zeta}{2} T\log N + o(\log N) & \text{if } m_1 = m - \frac{d}{2}, \end{cases} \quad (2)$$

where $\tau = \frac{1}{d}(m_1 - m) + \frac{1}{2}$,

$$\zeta = (2\pi)^{2(m_1-m)} \frac{\left(\int_{A^\theta(x,\xi)<1} dxd\xi\right)^{2m/d}}{\left(\int_{A_1(x,\xi)<1} dxd\xi\right)^{2m/d}}$$

and $A(x,\xi)$ is the principal symbol of the operator A. Then $\sqrt{\mathbb{I}_N}(\hat{\theta}_N - \theta)$ converges in distributions to a Gaussian random variable with zero mean and unit variance as $N \uparrow \infty$.

Recently, in the paper Kim and Park (2017), authors further develop the techniques, relied on the combination of Malliavin calculus (see, e.g., Nourdin & Peccati, 2012 and Nualart, 2006) and Stein's method (see, e.g., Chen, Goldstein, & Shao, 2011, Stein, 1972 and Stein, 1986), studied by Nourdin and Peccati in Nourdin and Peccati (2009a, b, 2010b, 2015), and apply these techniques to obtain the following optimal Berry–Esseen bound of CLT given in Theorem 1.

Theorem 2 (*Kim and Park*). *Assume that $m_1 \geq (3m/2) - (d/4)$. Then there exist constants $0 < c_{T,\theta,\beta} < C_{T,\theta,\beta} < \infty$, depending on θ, T and $\beta \in (0,1)$, such that, for sufficiently large $N \in \mathbb{N}$,*

$$c_{T,\theta,\beta} N^{-\frac{2m+d}{2d}} \leq \sup_{z\in\mathbb{R}} \left| \mathbb{P}(\sqrt{\mathbb{I}_N}(\hat{\theta}_N - \theta) \leq z) - \mathbb{P}(Z \leq z) \right| \leq C_{T,\theta,\beta} N^{-\frac{2m+d}{2d}}.$$

However, in the case when $m_1 < m - (d/2)$, Theorem 1 does not hold. In Hübner, Khasminskii, and Rozovskii (1993), authors investigate the asymptotic properties of the MLE θ_N^ϵ based on a finite-dimensional projection u_N^ϵ of the solution $u^\epsilon(t,x)$ $x \in [0,1]$ and $t \in [0,T]$, governed by SPDEs

$$du^\epsilon(t,x) = [\Delta u^\epsilon(t,x) + \theta u^\epsilon(t,x)]dt + \epsilon dW_Q(t,x), \quad t \leq T, \ x \in [0,1] \quad (3)$$

with the initial and boundary conditions given by

$$u^\epsilon(0,x) = f(x), \ \ f \in L_2(0,1), \quad (4)$$

$$u^\epsilon(t,0) = u^\epsilon(t,1) = 0 \ \text{ for all } \ 0 \leq t \leq T, \quad (5)$$

where $\Delta = \frac{\partial^2}{\partial x^2}$, Q is a nuclear covariance operator for the Wiener process $W_Q(t,x)$ taking values in $L^2(0,1)$. Let us set

$$\varphi_N^\epsilon(\theta) = \sum_{i=1}^N \frac{\lambda_i + 1}{2(\theta - \lambda_i)} \left\{ \nu_i^2 (e^{2(\theta-\lambda_i)T} - 1) - T\frac{\epsilon^2}{\lambda_i} \right\} \quad (6)$$

with $\lambda_i = (\pi i)^2$, $i \geq 1$ and $\nu_i = \int_0^1 f(x)\sin(i\pi x)dx$. Authors in Hübner et al. (1993) study asymptotic normality of $\frac{\sqrt{\varphi_N^0(\theta)}}{\epsilon}(\hat{\theta}_N^\epsilon - \theta)$ as $\epsilon \downarrow 0$ for fixed N, where the notation $\varphi_N^0(\theta)$ denotes the case when $\epsilon = 0$ in (6).

Afterward, in the paper Mishra and Prakasa Rao (2004), authors obtain the rate of convergence of asymptotic normality for the MLE $\hat{\theta}_N^\epsilon$ of parameter θ in SPDEs (3).

Theorem 3 (*Mishra and Prakasa Rao*). *There exists a constant C depending on θ, $\|f\|^2_{L^2(0,1)}$ and T such that for any $0 < \gamma < 1$ and $0 < \epsilon < 1$, depending on θ and T,*

$$\sup_{z\in\mathbb{R}} \left| \mathbb{P}\left(\frac{\sqrt{\varphi_N^\epsilon(\theta)}}{\epsilon} (\hat{\theta}_N^\epsilon - \theta) \leq z \right) - \mathbb{P}(Z \leq z) \right| \leq C\frac{\epsilon^\gamma(1+\sqrt{T})}{\epsilon^2 T + \|f\|^2_{L^2(0,1)}} + 3\sqrt{\epsilon^{1-\gamma}}, \quad (7)$$

where the random variable Z has the normal distribution with the zero mean and unit variance, and the normalizing factor $\varphi_N^\epsilon(\theta)$ is given in (6).

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