ARTICLE IN PRESS

Journal of the Korean Statistical Society ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of the Korean Statistical Society

journal homepage: www.elsevier.com/locate/jkss

Quantile based tests for exponentiality against DMRQ and NBUE alternatives

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ARTICLE INFO

Article history:
Received 23 May 2017
Accepted 17 December 2017
Available online xxxx

AMS 2000 subject classifications:
62G10
62G20

Keywords:
DMRQ
Exponential distribution
Gini mean difference
Mean residual quantile function
NBUE

ABSTRACT

Quantile-based reliability analysis has received much attention recently. We propose new quantile-based tests for exponentiality against decreasing mean residual quantile function (DMRQ) and new better than used in expectation (NBUE) classes of alternatives. The exact null distribution of the test statistic is derived when the alternative class is DMRQ. The asymptotic properties of both the test statistics are studied. The performance of the proposed tests with other existing tests in the literature is evaluated through simulation study. Finally, we illustrate our test procedure using real data sets.

1. Introduction

The exponential distribution is a very appealing model, particularly in life testing and reliability studies ever since the path-breaking paper by Epstein and Sobel (1953). For properties and applications of exponential distribution we refer to Galambos and Kotz (1978), Johnson, Kotz, and Balakrishnan (1994) and Marshall and Olkin (2007). The importance of the distribution is partly due to the remarkable property of constant hazard rate or equivalently constant mean residual life function and hence it is often used as a base line for evaluating families with non constant failure rates or mean residual life times.

Let X be non-negative continuous random variable with distribution function $F(x) = P(X \leq x)$. There are mainly two kinds of tests regarding the ageing criteria in reliability theory. Tests can be constructed to measure the departure from an ageing class to a different one or to check whether F follows exponential distribution against the alternative that F belongs to a particular ageing class. For a review on it, one can refer to Doksum and Yandell (1984, Ch. 26) and Henze and Meintanis (2005).

The works on problems related to tests for exponentiality against decreasing mean residual life (DMRL) and NBUE alternatives started from Hollander and Proschan (1975). Later, several researchers studied the same problem with different approaches. See Aly (1990), Anis (2013), Anis and Mitra (2011), Bergman and Klefsjö (1989), Bhattacharjee and Sen (1995), Jammalamadaka and Taufer (2002), Koul (1978), Lorenzo, Malla, and Mukerjee (2013) and Sankaran and Midhu (2016).

Modelling lifetime data using quantile functions has received much attention in recent years. Nair, Sankaran, and Vineshkumar (2012) explored the importance of Govindarajulu distribution as a lifetime model. Nair, Sankaran, and

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https://doi.org/10.1016/j.jkss.2017.12.003
1226-3192/

Balakrishnan (2013) studied the reliability properties of various models including Power-Pareto distribution, generalized lambda distribution of Ramberg and Schmeiser and four-parameter distribution of Van Staden and Loots. The main feature of above listed models is that their corresponding distributions are not in closed form. In such cases, where the quantile functions are only available to represent a distribution, quantile based criteria for ageing are needed and are listed in Nair and Vineshkumar (2011). Also while it comes to the testing exponentiality problems related to quantile function model, our usual distribution function based approach is not applicable. Strengthened by this reason, the present paper aims at proposing quantile-based test for exponentiality against DMRQ and NBUE alternatives.

In this context, problem of testing exponentiality against different alternatives in the quantile based framework has great significance. Janssen, Swanepoel, and Veraverbeke (2009) proposed a new test for exponentiality against new better than used in pth quantile of residual lifetime distribution. Fernández-Ponce and Rodríguez-Griñolo (2015) provided a test for exponentiality against NBUE class of distributions based on the excess wealth function and discussed its applications in environmental extremes. Sankaran and Midhu (2016) obtained a test for exponentiality against DMRQ or increasing mean residual quantile function (IMRQ) alternatives. However, we noted that the test proposed by Sankaran and Midhu (2016) is actually applicable to NBUE alternatives rather than DMRQ or IMRQ alternatives (see Remark 1). Motivated by this, we develop a test for exponentiality against DMRQ alternatives using mean residual quantile function. A test for exponentiality against NBUE alternatives using a characterization based on Gini mean difference is also proposed.

The article is organized as given below. In Section 2, we propose a Quantile-based test for exponentiality against DMRQ alternatives. The exact null distribution of the test statistic is derived. The asymptotic properties of the test statistics are also studied. In Section 3, we develop a quantile based test for exponentiality against NBUE alternatives using a characterization based on Gini mean difference. The asymptotic properties of the test statistic are also presented. The result of the simulation study and real data analysis are reported in Section 4. Finally in Section 5, we give conclusions of our study.

2. Test for exponentiality against DMRQ

For right continuous function F, the corresponding quantile function is defined as

$$Q(u) = \inf\{x \;:\; F(x) \geq u\}, \quad 0 \leq u \leq 1.$$

If F is continuous, then we have a unique inverse $Q(u) = F^{-1}(u)$. Let $F_n(.)$ be the empirical distribution function defined as $F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \leq x)$, where I denotes the indicator function and $X_1, X_2, \ldots, X_n$ are independent random sample of size n; from F. The non-parametric estimator of $Q(u)$ is given by

$$\widehat{Q}(u) = \inf\{x \;:\; F_n(x) \geq u\}, \quad 0 \leq u \leq 1.$$

Analysis based on remaining lifetime of an object is intuitively more appealing than the popular hazard function. Expected value of the remaining lifetime is extensively used to study the ageing behaviour of a component or a system. The mean residual life function is defined as

$$m(x) = E(X - x|X > x) = \frac{1}{(1 - F(x))}\int_x^{\infty} y(1 - F(y))dy.$$

Based on the random sample $X_1, X_2, \ldots, X_n$; from F, a non-parametric estimator of mean residual life function at x is given by

$$\widehat{m}(x) = \frac{\sum_{i=1}^{n}(X_i - x)I(X_i > x)}{\sum_{i=1}^{n} I(X_i > x)},$$

where $I(X_i > x) = 1$, if $X_i > x$ and 0 otherwise. Next we give the definition of mean residual quantile function.

Definition 1. Let X be non-negative random variable with continuous distribution function $F(.)$ and quantile function $Q(.)$, the mean residual quantile function is defined as (Nair et al., 2013)

$$M(u) = \frac{1}{1-u}\int_u^1 (Q(p) - Q(u))\, dp. \tag{1}$$

The $M(u)$ can be interpreted as the average remaining life beyond the $100(1-u)\%$ of the distribution.

A non-parametric estimator of $M(u)$ is given by (Sankaran & Midhu, 2016)

$$\widehat{M}(u) = \frac{1}{1-u}\int_u^1 \left(\widehat{Q}(p) - \widehat{Q}(u)\right) dp.$$

Based on the random sample $X_1, X_2, \ldots, X_n$; from F, using the definition of $\widehat{Q}(u)$ we have

$$\widehat{M}(i/n) = \frac{1}{(n-i)}\sum_{j=i+1}^{n}\left(X_{(j)} - X_{(i)}\right), \quad i = 1, 2, \ldots, n-1. \tag{2}$$