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Optimal number of components in a load-sharing system for maximizing reliability

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ABSTRACT

A k-out-of-n:G load sharing system is a cluster of n components designed to withstand a certain amount of load in field operation, working only if no fewer than k components work. Previous research on a load sharing system has focused on predicting the time-independent reliability from the stress–strength model or estimating the unknown parameters of the time-dependent reliability for a given load sharing rule. Differently, in this paper, we consider the problem of determining the optimal n to maximize the reliability of both n-out-of-n:G and $(n-1)$-out-of-n:G load sharing systems. Since the load of each component decreases in n, the proportional hazard model is employed to relate the component failure rate with the load, assuming that the components, which have exponential distributions for given loads, are independent of each other. We then derive a sufficient condition under which a smaller number of components each withstanding a high load is preferred to a larger number of components each withstanding a small load. A numerical example is given for the rocket propulsion system to illustrate the result.
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1. Introduction

In the early design stage, a designer rates a system to withstand a certain amount of load during operation. Options may be available to develop one large component carrying the entire design rated load or a cluster of several small components each sharing the same load. When a component is operating at its design rated load, the corresponding failure rate is called the nominal failure rate (Bazovsky, 2004; Kim, Roh, Lee J.-W, & Zuo, 2016). Empirical studies have observed that the component failure rate strongly depends on the design rated load, and thus a reduced load results in a lower nominal failure rate, or equivalently, a higher reliability (Misra, 2008). As the number of components in a system increases, the design rated load of each component decreases, implying that the failure rate of each component gets smaller. However, the overall reliability of the entire system decreases considerably as the number of components in the system increases because one component failure results in the failure of the entire system. Therefore, an additional component is used in the system only if it results in reducing the failure rate of each component such an extent that it offsets the increase in overall system failure rate.

To improve the system reliability further, a designer often implements a component-out-design in which the load of a failed component is redistributed equally among surviving components, which continue to complete the mission. The component-out-design can be considered as a k-out-of-n:G load sharing system for $k \leq n$, where n is the total number of components in the system and k is the minimum number of working components required for the system to work. Since the system needs to withstand the design rated load when a total of k components work, the difference between n and k represents the redundancy. The series system is a special case of k-out-of-n:G load sharing system where k is given by n.

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In practice, k-out-of-n:G load sharing systems have a wide range of applications in fields such as textile engineering, material science, and nuclear reactor safety (Misra, 2008). Especially, in the design of the propulsion system in a launch vehicle to generate a required thrust, we may choose to use a smaller number of engines, each with a high thrust output, or a larger number of components, each with a small thrust output. For example, four engines are clustered in the first stage of Korea Space Launch Vehicle II (KSLV-II), whereas eight engines are clustered in the first stage of the Saturn I rocket. Furthermore, the engine-out-design is implemented, in which the surviving engines share the thrust of the failed engine for completing the mission in case one engine shuts down safely, without causing the entire cluster failure (Sharfie, Cheragh, Dashti, Zaretalab, & Fakhredaie, 2015).

Previous researches in load sharing systems have focused on predicting the time-independent reliability based on the stress strength model (Daniels, 1945; Durham & Lynch, 2000; Harlow, Smith, & Taylor, 1983; Lee, Durham, & Lynch, 1995; Smith, 1982), or estimating the unknown parameters of time-dependent reliability for a given load sharing rule using a lifetime distribution such as an exponential distribution (Lin, Chen, & Wang, 1993; Yinghui & Jing, 2008), Weibull distribution (Kececioglu & Jiang, 1991; Lynch, 1999), Lindely distribution (Singh & Gupta, 2012), or a general distribution (Huang & Xu, 2010; Kim & Kvam, 2004; Kvam & Pena, 2005; Park, 2013). The optimal design problem has been considered only for k-out-of-n:G system, rather than for a load sharing system, after assuming that the component reliability is constant regardless of k and n (Pham, 1992a, b; Suich & Patterson, 1991). In a load sharing system, the design rated load of each component depends on the selected values of k and n, and thus the component reliability depends on the values of k and n. Thus far, no research has been done to investigate whether a cluster of small components gives a higher reliability than a cluster of large components, when developing a system to withstand a certain amount of load.

This paper intends to determine the optimal number of components to maximize the reliability of a k-out-of-n:G load sharing system, assuming that k is given by n or $n-1$. If k is given by n, there is no redundancy, and one component failure results in immediate system failure. Alternatively, if k is given by $n-1$, then one component is redundant so that the load of a failed component can be redistributed equally to the remaining $n-1$ components. For each case, we first present a model to calculate the system reliability in terms of n, assuming that the components in the cluster are independent of each other and have exponential distributions for given design rated loads. In addition, the proportional hazard rate model is employed to relate the component failure rate with the design rated load. Then, we investigate whether a cluster of several components gives a higher overall reliability than one large component. A numerical example for the rocket propulsion system is given to illustrate the model.

2. *n*-out-of-*n* load sharing system

In this section, we first explain how to use the proportional hazard model for relating the designed rated load of a component with its failure rate. Then, we calculate the reliability of the n-out-of-n load sharing system for determining the optimal n required to maximize the reliability.

Consider the design of a component to withstand a certain amount of load during operation. The component failure rate strongly depends on the design rated load as well as the operational load. Assuming that a component is operating at its design rated load, the corresponding failure rate is called the nominal failure rate (Bazovsky, 2004; Kim et al., 2016). When comparing design alternatives in the early design stage, the component lifetime is typically assumed to follow an exponential distribution, or equivalently, the nominal failure rate is constant with time (Kim, 2013; Kim & Zuo, 2015). Suppose that the nominal failure rate of a given component to withstand a design rated load of L is known from previous experience as λ. Then, λ is used as the baseline failure rate to predict the nominal failure rate of a new component having a different design rated load. Let λ_0 be the nominal failure rate of a new component having a design rated load of L_0. Empirical studies have proved that a reduced load results in a lower failure rate or a higher component reliability (Misra, 2008). This means that $\lambda_0 < \lambda$ if $L_0 < L$. To predict λ_0 from λ, we employ Cox's proportional hazard model (Cox, 1972; Misra, 2008), in which the failure rate of a component is the product of the baseline failure rate and a multiplicative factor, which is a function of the load on the component. If we further assume that the multiplicative factor follows a power law, then we can express

$$\lambda_0 = \lambda \left(\frac{L_0}{L} \right)^{\delta}, \qquad \delta > 0, \tag{1}$$

where δ is the power parameter for scaling different design rated loads. The parameter δ in Eq. (1) depends on the ability of the system developer to produce a component that can withstand a different value of load. The nominal failure rate for different values of δ is shown in Fig. 1 with a baseline load L. A large value of δ indicates a fast decrease in the failure rate as the design load decreases from the baseline load, and a fast increase in the failure rate as the design load increases from the baseline load.

Consider the reliability of n-out-of-n load sharing system for any n where $n \geq 1$. Since there is no redundancy, one component failure results in immediate system failure. Given that the design rated load of the entire system is L_1, suppose that one large component is developed to carry L_1. Let λ_1 and $R_1(t)$ denote the corresponding nominal failure rate and reliability for a fixed mission time $t > 0$, respectively. Then,

$$R_1(t) = \exp(-\lambda_1 t), \qquad \lambda_1 > 0, \quad t > 0.$$

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