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Bayesian empirical likelihood methods for quantile comparisons

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ABSTRACT

Bayes factors, practical tools of applied statistics, have been dealt with extensively in the literature in the context of hypothesis testing. The Bayes factor based on parametric likelihoods can be considered both as a pure Bayesian approach as well as a standard technique to compute p-values for hypothesis testing. We employ empirical likelihood methodology to modify Bayes factor type procedures for the nonparametric setting. The paper establishes asymptotic approximations to the proposed procedures. These approximations are shown to be similar to those of the classical parametric Bayes factor approach. The proposed approach is applied towards developing testing methods involving quantiles, which are commonly used to characterize distributions. We present and evaluate one and two sample distribution free Bayes factor type methods for testing quantiles based on indicators and smooth kernel functions. An extensive Monte Carlo study and real data examples show that the developed procedures have excellent operating characteristics for one-sample and two-sample data analysis.

1. Introduction

The use of Bayes Factors (BF) is commonly cited as an efficient approach to incorporating external information into the inference process about a given hypothesis of interest when the likelihood functions have parametric forms (e.g., Kass, 1993; Kass & Raftery, 1995). Although this method can be employed as a pure Bayesian alternative to frequentist techniques based on p-values, BF type statistics can also serve statistical decision making in the terms of traditional test statistics with controlled distributions under the null hypothesis, via, e.g., the use of theoretical asymptotic propositions. In this article, we propose to replace the parametric likelihoods in BFs with empirical likelihoods (Owen, 2001), constructing nonparametric BF type procedures.

In particular, we focus here on the development of BF based procedures for nonparametric testing about the quantiles in both the one- and two-sample settings.

The following practical issues motivate the work regarding the development of novel tests for quantiles. In biomedical research, estimates of quantiles, e.g., median survival times, are frequently used to characterize outcome variables. For heavily censored data, summary measures based on sample quantiles are generally preferable to the estimated mean survival since they have smaller bias. Moreover, the difference in treatment effects might be estimated using the difference between quantiles under a location-shift assumption (e.g., Hughes, 2000). Note that one of the main goals in applied research is to determine whether two independent groups (e.g., treatment and control groups) differ, and, if they do differ, to quantify the

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magnitude of this effect. Wilcox (1995) provided many two-sample test examples showing that evaluations and comparison of quantiles led the investigators to more satisfactory conclusions in terms of relative power compared to tests based on sample moments. Sometimes, the populations of the two groups differ not only by location but also by the shape and scale of the distribution. In such cases, (1) comparing quantiles, e.g., the medians may provide different information than comparing sample moments of the observations, e.g., the means, and (2) permutation based tests for differences in quantiles in this setting would violate the exchangeability assumption under the null hypothesis (e.g. Hutson, 2007 or examples involving medians). An additional benefit of using sample quantiles in the two-group setting is that they provide a measure of robustness relative to their respective influence functions as compared to moment based tests (e.g. Yu, Vexler, Hutson, & Baumann, 2014; Yu, Vexler, Kim, & Hutson, 2011). Similarly, the quantile approach is feasible and useful when tests based on moments are not. For example, when estimating the parameter of a Cauchy distribution the sample mean is not a consistent estimator of the location parameter while the sample median is (e.g., Zhou & Jing, 2003). Hughes (2000) emphasizes that in the context of censored measurement studies, the non-parametric approach, e.g., testing the median, is more robust to the censored data than the parametric approach based on moments. Yu et al. (2011) argue the relevance of median evaluation of biomarkers in the context of cancer research, such as enzyme-linked immunosorbent assay (ELISA) and Western blot analysis.

In certain studies, we may have prior information regarding the values of quantiles before evaluating a given dataset. In this article, we consider quantile evaluation conditional on prior information and observations, adopting well-developed Bayesian methods, specifically hypothesis testing based on BF mechanics. This approach provides practical and flexible tools for combining information and partial pooling of inferences, according to parametric likelihood principles (e.g., Carlin & Louis, 2008). BF type procedures are often discussed in the context of criticism of the traditional p-value approach, since the Bayesian approach to hypothesis testing can be shown to be much simpler and more sensible in principle in a variety of situations (e.g., Kass & Raftery, 1995; Carlin & Louis, 2008, p. 39). Furthermore, asymptotic propositions related to analyzing null distributions of BF type statistics provide the possibility of considering traditional testing rules based on these statistics (e.g., Kass & Wasserman, 1995). The asymptotic results also simplify computations of BF based statistics that often consist of complex integrals, marginal distributions, calculated by numerical methods. Within this framework we will consider the stated problems non-parametrically, whereas the classical Bayesian methodology involves parametric likelihoods.

Nonparametric tests offer a simple and reliable statistical technique in applications. They possess the desirable property of having the same sampling distribution for all continuous distributions. For the two-sample location shift setting commonly used nonparametric procedures include the Wilcoxon normal scores and median (Mood, 1954) tests. Results regarding the median test are varied. A number of authors (e.g., Freidlin & Gastwirth, 2000) demonstrate the poor performance of the median test in very small samples, as well as the loss of power of the median test relative to the Wilcoxon test in the case of highly unbalanced samples. Yu et al. (2011), on the other hand, note that the median test is valid under weaker conditions than other rank tests. Results of Yu et al. (2011) show that testing for two-group medians based on the empirical likelihood (EL) approach is workable even with the violation of the exchangeability assumption under the null hypothesis (Hutson, 2007). This article develops different tests for quantiles via the EL methodology.

The EL approach (e.g., Lazar & Mykland, 1999; Owen, 2001; Vexler, Liu, Kang, & Hutson, 2009; Yu, Vexler & Tian, 2009) is a nonparametric method of statistical inference, which allows researchers to use traditional likelihood methodology without having to assume known forms of data distributions. The EL technique is very effective because it inherits many of the characteristics of parametric likelihood methods. This method is widely used to find efficient estimators and to construct tests with good power properties. The EL method is also very flexible. For this article's purposes, we refer to the research shown in Lazar (2003), which demonstrates that the EL is a valid function for Bayesian inference. To present this conclusion, Lazar (2003) uses the Monahan & Boos heuristic (e.g., Monahan & Boos, 1992) and examines the frequentist properties of Bayesian intervals. Thus, Lazar (2003) explores a simple way of proposing nonparametric Bayesian inference via an application of the standard theory with a prior on the functional of interest and EL taking the role of a model-based likelihood.

In this article, the methods mentioned above are applied to develop distribution free quantile testing procedures. The proposed technique is used for one-sample and two-sample analysis. We evaluate different mechanisms based on indicators and smooth kernel functions. The results we obtain can also be considered as certain extensions of the Bayesian nonparametric estimations of medians presented in Doss (1985a, b) and the EL estimation proposed in Chen and Hall (1993). Note that proof schemes used in this article can be of independent interest to investigators dealing with applications of adapted Laplace methods (e.g., Bleistein & Handelsman, 1975; Davison, 1986; Kass & Raftery, 1995) to handle step functions and EL functions. These proof schemes allow us to obtain asymptotic results similar to those of parametric BFs.

The paper is organized as follows. In Section 2, we outline the relevant ideas of Bayes Factors and empirical likelihood. These methods are combined to develop the one and two sample EL BF inference for quantiles. In Section 3, we carry out an extensive Monte Carlo study to evaluate the new procedures. In Section 4 we present data examples that demonstrate the excellent applicability of the proposed method in practice. Finally, in Section 5 we conclude by summarizing the most important points related to this research.

2. Method

In this section, we first outline basic techniques related to: (1) BF type tests; (2) ELs; (3) Bayesian ELs. The core of our inference procedure for quantiles is to combine these three.