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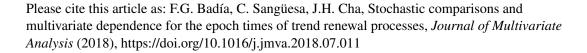
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Stochastic comparisons and multivariate dependence
for the epoch times of trend renewal processes

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Abstract

In this paper we study stochastic comparisons and dependence properties for the epoch and inter-epoch times of trend renewal processes with different trend functions. These results extend some of the results on the non-homogeneous Poisson process in Belzunce et al. [5, 7] to trend renewal processes. Some applications of the results to a shock model, a repair process, and a specific class of intermediate order statistics are provided.

1. Introduction

Non-homogeneous Poisson processes (NHPP) and renewal processes (RP) are the two most commonly used models in the analysis of recurrent failure data as the effect of minimal and perfect repairs are modeled by those processes, respectively; see, e.g., [14, 22, 36]. A trend-renewal process (TRP) which includes the NHPP and RP as special cases was introduced in [23, 24]. The TRP model has been widely used; see, e.g., [12, 31, 34, 35]. The nonparametric maximum likelihood estimator of the intensity or trend function for the TRP was derived in [13].

The stochastic comparison of various aging measures associated with lifetimes has been an interesting and important topic in reliability theory and related fields. A useful tool employed in such studies is the theory of stochastic orders. Stochastic comparisons of system lifetimes, mixture distributions, order statistics, etc., have been studied intensively in the literature. Some recent surveys on the topic are [17, 27, 28] (for system lifetimes), [20] (for mixture distributions), [29] (for generalized mixtures), [1, 5] (for counting processes), and [6, 21] (for order statistics).

Concepts of association or dependence play a central role when considering multiple random sources in statistical models as they describe the relationship between two or more random variables. A classical monograph on multivariate dependence concepts is [15]. There is a vast literature on multivariate dependence properties in applied probability that focuses on reliability theory. Some interesting applications can be found in [19] (for multivariate mixtures), [9, 18] (for order statistics), [1, 7] (for counting process), and so on.

In [5] a comprehensive stochastic comparison of the epoch times and inter-epoch times of NHPP processes and non-homogeneous pure birth processes (NHPBP) with different intensity functions is developed. Following [5], this paper reports results for the stochastic comparison of the epoch and inter-epoch times of TRP processes; see Theorems 2–3. Furthermore, we provide multivariate dependence concepts and aging properties of the epoch and inter-epoch times of TRP (see Theorem 1) in the spirit of [7] that analyzes multivariate aging properties of epoch times of NHPP. Analogous stochastic comparisons and multivariate dependence concepts have been addressed in [1] for the case of generalized Pólya processes [10] with different intensity functions. Stochastic comparisons between NHPP with different intensity functions in the increasing convex order (which is not considered here) are provided in [8], whereas the increasing directionally convex order is considered in [3].

The general framework for this paper is as follows. In Section 2, the notation and the main tools used are introduced. In Section 3, stochastic comparisons of the epoch and inter-epoch times of trend renewal process with the

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