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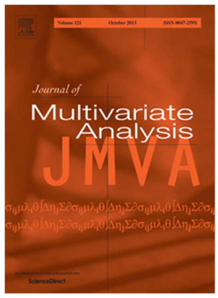

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Membership testing for Bernoulli and tail-dependence matrices

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Abstract

Testing a given matrix for membership in the family of Bernoulli matrices is a long-standing problem; the many applications of Bernoulli vectors in computer science, finance, medicine, and operations research emphasize its practical relevance. After the three-variate case was covered by Chaganty and Joe [3] by means of partial correlations, a novel approach towards this problem was taken by Fiebig et al. [9] for low-dimensional settings, i.e., $d \leq 6$. The latter authors were the first to exploit the close relationship between the probabilistic world of Bernoulli matrices and the well-studied correlation and cut polytopes. Inspired by this approach, we use results from [6, 7, 9] in a pre-phase of a novel algorithm to check necessary as well as sufficient conditions, before actually testing a matrix for Bernoulli compatibility. In our main approach, however, we build upon an early attempt by Lee [13] based on the vertex representation of the correlation polytope and directly solve the corresponding linear program. To deal appropriately with the issue of exponentially many primal variables, we propose a specifically tailored column generation method. A straightforward, yet novel, analysis of the arising subproblem of determining the most violated dual constraint in the column generation process leads to an exact algorithm for membership testing. Although the membership problem is known to be NP-complete, we observe very promising performance up to dimension $d = 40$ on a broad variety of test problems. An important byproduct of the numerical solution is a representation for Bernoulli matrices with (only) d^2 many vertices that gives rise to an efficient simulation routine.

Keywords: Bernoulli-compatible matrix, Binary quadratic programming, Column generation, Tail-dependence matrix
2000 MSC: 60-08, 62H20, 52B12, 90C08

1. Introduction and motivation

1.1. Motivation

Characterizing a correlation matrix in terms of its algebraic properties is classical content of an introductory course to multivariate statistics. The closely related question, however, namely testing if a matrix $B \in \mathbb{R}^{d \times d}$ is a Bernoulli matrix or a matrix of pairwise tail-dependence coefficients is much harder. Literature related to this problem is spread over different communities ranging from probability and operations research to applications in various disciplines; see, e.g., [15] and other references listed therein. This results in an inconsistent notation and nomenclature that makes it challenging to keep track of all relevant results. Our original interest in the problem stems from a probabilistic treatment by Embrechts et al. [7], research inspired by an actuarial application, which ends with the statement: "Concerning future research, an interesting open question is how one can (theoretically or numerically) determine whether a given arbitrary nonnegative, square matrix is a tail-dependence or Bernoulli-compatible matrix. To the best of our knowledge there are no corresponding algorithms available." From a methodological point of view, however, much closer is the deep mathematical investigation of the geometry of the problem by Fiebig et al. [9] who succeed in characterizing low-dimensional cases (i.e., $d \leq 6$) in terms of an analysis of the geometry of the closely related cut polytope.

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