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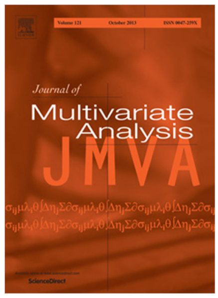

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The joint distribution of the sum and maximum of dependent Pareto risks

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Abstract

We develop a stochastic model for the sum X and the maximum Y of dependent, heavy-tail Pareto components. Our results include explicit forms of the probability density and cumulative distribution functions, marginal and conditional distributions, moments and related parameters, parameter estimation, and stochastic representations. We also derive mixed conditional tail expectations, $\mathrm{E}(X|Y > y)$ and $\mathrm{E}(Y|X > x)$, which provide measures of risk frequently used in finance and insurance. An extension incorporating a random number N of components in the sum and the maximum, along with its basic properties, is included as well. Two data examples from finance illustrate modeling potential of these new multivariate distributions.

Keywords: Clayton copula, Common background risk, Dependence by mixing, Generalized Pareto distribution, Risk measures, Tail conditional expectation

1. Introduction

In this paper we present a theory of a new joint probability distribution describing the random vector

$$(X, Y) \stackrel{d}{=} \left(\sum_{i=1}^{n} X_i, \bigvee_{i=1}^{n} X_i \right), \tag{1}$$

consisting of the sum and the maximum of $n \in \mathbb{N} = \{1, 2, \ldots\}$ identically distributed and dependent components of a random vector $\mathbf{X} = (X_1, \ldots, X_n)$ given by the joint probability density function (pdf) defined, for all $x_1, \ldots, x_n \in \mathbb{R}_+$, by

$$f_{\alpha,\beta,n}(x_1, \ldots, x_n) = \frac{(\alpha\beta)^n \Gamma(1/\alpha + n)}{\Gamma(1/\alpha)} \left(1 + \alpha\beta \sum_{i=1}^{n} x_i \right)^{-1/\alpha - n}, \tag{2}$$

where $\beta > 0$ is a scale parameter and $\alpha \geq 0$ is a tail parameter. The univariate margins of $\mathbf{X}$ are Lomax, also known as Pareto Type II, distributions with survival function given, for all $x \in \mathbb{R}_+$, by $\Pr(X_i > x) = (1 + \alpha\beta x)^{-1/\alpha}$; see, e.g., [10, 59]. For this reason, the distribution of $\mathbf{X}$ is known in the literature as the multivariate Lomax or Pareto (Type II); see, e.g., [10, 47, 58, 67, 79, 84, 85]. In this multivariate setting, Model (2) is often described as one with marginals dependent by mixture or by mixing (see, e.g., [79]), due to the stochastic representation

$$\mathbf{X} = (X_1, \ldots, X_n) \stackrel{d}{=} (E_1/Z, \ldots, E_n/Z), \tag{3}$$

where $\mathbf{X}$ is multivariate Lomax with pdf (2) and $E_1, \ldots, E_n$ are independent and identically distributed (iid) exponential variables with common pdf defined, for all $x \in \mathbb{R}_+$, by $f(x) = \beta e^{-\beta x}$. The quantity Z that appears on the

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