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Juan L. Padilla, Caio L.N. Azevedo, Victor H. Lachos

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Multidimensional multiple group IRT models with skew normal latent trait distributions

Juan L. Padilla[a], Caio L.N. Azevedo[a,*], Victor H. Lachos[b]

[a]*Department of Statistics, University of Campinas, Brazil*
[b]*Department of Statistics, University of Connecticut, Storrs, CT 06269, USA*

Abstract

Multidimensional item response theory (MIRT) models are quite useful to analyze data sets involving multiple skills or latent traits, which occur in many applications. However, most of the works consider the usual multivariate (symmetric) normal distribution to model the latent traits and do not deal with the multiple group framework. Also, in general, the works consider a limited number of model fit assessment tools and do not investigate the measurement instrument dimensionality in a detailed way. When the assumption of normality of the latent traits distributions does not hold, misleading results and conclusions can be obtained. Our goal is to propose a MIRT multiple group model with multivariate skew normal distributions under the centered parameterization to model the distribution of the latent traits of each group, presenting simple and feasible conditions for model identification. Such an approach is more flexible than the usual multivariate (symmetric) normal one. In addition, a full Bayesian approach for parameter estimation, structural selection (model comparison and determination of the dimensionality of the measurement instrument) and model fit assessment are developed through Markov Chain Monte Carlo algorithms. The proposed tools are illustrated through the analysis of a real data set related to the first stage of the University of Campinas 2013 admission exam.

Keywords: Bayesian inference, Centered parameterization, Item response theory, MCMC algorithms, Model fit assessment, Multivariate skew normal distribution

1. Introduction

Item response theory (IRT) models are one of the most important sets of psychometric tools for data analysis. Their applicability goes from educational assessment to biological essays. IRT models combine, at least, two sets of unknown quantities: the latent traits (person parameters) and item parameters related to measurement instruments of interest, e.g., a cognitive test, genetic experiments or a psychiatric questionnaire. A common assumption for IRT models is that the latent traits follow symmetric normal distributions with possibly different population parameters, when a multiple group or a multilevel structure is considered. Deviations from the normality of the latent traits distributions may be related to the presence of asymmetry and/or heavy tails and/or multimodality. In the unidimensional IRT (UIRT) context, it is possible to find examples of data sets for which the assumption of normality is not reasonable due to some of these characteristics; see [4, 14, 44]. The lack of normality or even symmetry, in a broader sense, for the latent traits distributions, can lead to biased estimates; see [3, 20, 21, 45] and references therein.

Multidimensional item response theory (MIRT) comprises a set of statistical models which are useful to analyze data sets involving multiple skills or latent traits, which occurs in many of the applications. Most of the works in the literature do not consider or only consider a limited number of model fit assessment tools, and do not investigate the measurement instrument dimensionality in a detailed way, while also dealing with the model nonidentiability in a different way than that we presented here and only for the one-group model. The assumption of multivariate (symmetric) normality is also considered for other classes of IRT models with multiple latent traits (with some underlying correlation structure), as in [1, 9], which are examples of longitudinal IRT models.

*Corresponding author
Email address: cnaber@ime.unicamp.br (Caio L.N. Azevedo)