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Equivalence and orthogonality of Gaussian measures on spheres

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Abstract

The equivalence of Gaussian measures is a fundamental tool to establish the asymptotic properties of both prediction and estimation of Gaussian fields under fixed domain asymptotics. The paper solves Problem 18 in the list of open problems proposed by Gneiting [24]. Specifically, necessary and sufficient conditions are given for the equivalence of Gaussian measures associated to random fields defined on the d-dimensional sphere $\mathbb{S}^d$, and with covariance functions depending on the great circle distance. We also focus on a comparison of our result with existing results on the equivalence of Gaussian measures for isotropic Gaussian fields on $\mathbb{R}^{d+1}$ restricted to the sphere $\mathbb{S}^d$. For such a case, the covariance function depends on the chordal distance being an approximation of the true distance between two points located on the sphere. Finally, we provide equivalence conditions for some parametric families of covariance functions depending on the great circle distance. An important implication of our results is that all the parameters indexing some families of covariance functions on spheres can be consistently estimated. A simulation study illustrates our findings in terms of implications on the consistency of the maximum likelihood estimator under fixed domain asymptotics.

Keywords: Chordal distance, Equivalence of Gaussian measures, Great circle distance, Positive definite functions, Schoenberg coefficients.
2010 MSC: 28C20, 42A82, 42A16, 86A32,

1. Introduction

1.1. Motivation

Statistical analysis of processes defined over the entire globe has attracted a lot of attention in recent years and as a result, the literature on Gaussian random fields defined over spheres is becoming ubiquitous in areas as diverse as mathematical analysis [3, 26, 40, 41], probability theory [2, 16, 27, 36], spatial point processes [42], spatial geostatistics [22, 28, 29], space-time geostatistics [4, 15, 44], and mathematical physics [32, 37, 38]. Global models, especially for climate data, are also finding applications in many fields; see, e.g., [9, 10, 12, 44], and references therein.

The equivalence of Gaussian measures [30, 47] represents an essential tool to establish the asymptotic properties of both prediction and estimation of Gaussian fields defined on Euclidean spaces, under fixed domain (also called infill) asymptotics. Such a framework typically applies when more and more data are collected by sampling densely in a bounded set. Notable examples of application of the equivalence of Gaussian measures to fixed domain asymptotics for prediction can be found in [50, 53, 54]. The equivalence of Gaussian measures has been used in [57], and more recently in [5], for specific parametric families of covariance functions.

Fixed domain asymptotics is a natural framework for studying Gaussian random fields defined on d-dimensional spheres. However, the literature on conditions for the equivalence of Gaussian measures associated to stochastic processes over d-dimensional spheres has been sparse. We quote verbatim from Problem 18 in [24] as following

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