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Technical Note

Practical improvements to real and imaginary spectral based modal parameter measurements of SDOF systems

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ABSTRACT

For a single degree of freedom system, especially with non-light damping, the use of the real spectral part of either the displacement or velocity responses (or the transfer functions based on them) has the advantages of determining the natural frequency (f_n) directly, independent of the response parameter, and providing an accurate measurement for damping (ζ). However, this method is sensitive to spectral phase errors due to temporal misalignment of the signal or due to net inter-channel delay differences caused by signal filtering. Two techniques are presented to correct for these errors; one based on the correct temporal alignment of the impulsive part, and the other on the infimum of the imaginary spectral part. These are first demonstrated using a numerical model, and are shown to facilitate the correct measurement of f_n and bring ζ within the expected error limits due to quantisation. Secondly, they are applied to an experimental system and are seen to greatly improve the consistency between measurements using different methods.

1. Introduction

The modal natural frequency (f_n) and damping (ζ) are two parameters of a system commonly required to be estimated. For a single degree of freedom (SDOF) system popular estimation methods include: peak picking (PP [1–5]), real and imaginary part methods (Re-P [2] and Im-P [4]) and the Nyquist method [2,4,5] (that evaluates a frequency from the rate of sweep around the circle on the Argand plane). These are straightforward methods that allow the parameters to be estimated directly from specific frequencies, whilst more complex methods exist that rely on, for example, linear interpolation or circle-fitting [2,5]. Here the Re-P and Im-P methods are the focus of the study as they either have advantages over other methods and/or have useful properties allowing errors to be reduced. After implementing the error reduction technique described, more complex (and potentially more accurate) estimation methods could be applied if desired.

In order to measure modal parameters accurately, especially for non-lightly damped systems, each method must be pertinently applied to a spectrum based on either the velocity (V) or displacement (X) spectral response. The Re-P method determines f_n from the latter response [2], however, it is shown that this is also true for the V-based and acceleration-based spectra. Ref. [4] diagrammatically suggests that f_n can be estimated using the Im-P method for an X-based spectrum, it is shown that f_n can be determined exactly from a V-based spectrum for a viscously damped system. The Re-P method additionally allows the system damping to be accurately determined for systems with both light and heavy damping. Despite the advantages of the Re-P and the Im-P methods, these methods are sensitive to differences in the signal processing (*e.g.* filters) on the response and excitation measurement channels or, when processing a single-channel impulsive-excitation response, incorrect temporal alignment of the response signal.

Fig. 1 shows a typical measurement system with the response ($x(t)$ or $v(t)$) and excitation ($p(t)$) time signals as inputs. The modal parameters can be measured using either 1- or 2-channel measurements. For the latter the response signal spectra (X and V) are combined with the excitation signal (P) to form frequency response functions (FRFs). In this case, both channels have a common temporal alignment which is mutually compensated when the FRF is generated, however the spectrum will still be prone to phase errors due to differences in channel filtering. In some cases the excitation signal may not be available, and 1-channel measurements are possible for impulsive excitation. The pulse width should be sufficiently small so that its energy is (practically) constant over the bandwidth of interest. Here the impulsive part of the acceleration response (which usually has an asymmetric bell-shaped form) must be correctly aligned with the time origin to produce a spectrum without phase error. In this case a scaled FRF results, but still allows the modal parameters to be evaluated. The theoretical

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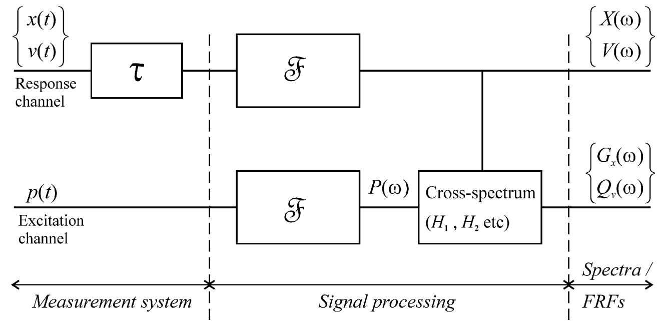

Fig. 1. Two-channel measurement system with temporal signal inputs, differential phase function (τ) and the conversion to the frequency domain by the Fourier transform ($\mathscr{F}$).

development uses the FRFs, but can be validly applied to the corresponding response parameter spectra alone.

Here the effect of these errors on the determination of f_n and ζ is studied for a viscously damped SDOF system, and then two techniques are introduced to improve the accuracy of measurements under such conditions. The PP method is seen to determine f_n from a V-based spectrum and this is used as a reference measure for experimental results since it is unaffected by phase.

In this technical note the measured/estimated values of f_n and ζ are distinguished from the actual values using an over hat symbol. Here a notation is used to succinctly represent the parameter evaluated as a function of a measurement method applied to a specified spectrum. For example, $\hat{f}_n(\mathrm{Re}\{X\})$ represents the estimate of f_n measured by applying the Re-P method to the displacement spectrum. Also, the circular frequency (f) and angular frequency ($\omega = 2\pi f$) are used interchangeably as appropriate, but when used with the same subscript are understood to refer to the same frequency.

2. Measuring modal parameters from the magnitude, and the real and imaginary parts of the spectrum

The FRF of a SDOF system when expressed as displacement normalised by the applied force, G_x, is [2–4]

$$G_x(\omega) = \frac{1}{m}\frac{1}{\omega_n^2 - \omega^2 + 2\mathrm{j}\zeta\omega_n\omega}, \tag{1}$$

where m is the rigid system mass, which is considered to be unity in the examples given below using the numerical model. The natural frequency (ω_n) and the damping (ζ) are defined from the system parameters as

$$\omega_n = \sqrt{\frac{k}{m}},\ \zeta = \frac{c}{2\omega_n\omega}, \tag{2a, b}$$

where c is the viscous damping coefficient and k the spring constant of the system. The FRFs for the velocity and acceleration responses (G_v and G_a) can be calculated by from the generalized derivative property of the Fourier transform [6]:

$$G_v(\omega) = \mathrm{j}\omega G_x(\omega),\ G_a(\omega) = -\omega^2 G_x(\omega). \tag{3a, b}$$

To maintain the same spectral response characteristics of real and imaginary parts in all of the response parameters as for the G_x spectrum, quadrature corrected versions of G_v and G_a are used, Q_v and Q_a, respectively, which are

$$Q_v(\omega) = \omega G_x(\omega),\ Q_a(\omega) = \omega^2 G_x(\omega). \tag{4a, b}$$

2.1. Magnitude of the Q_v spectrum

Using (1), (3) and (4) the magnitude of Q_v is found to be

$$|\ Q_v(\omega)\ | = \frac{1}{m}\frac{\omega}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}. \tag{5}$$

Setting $\mathrm{d}(|Q_v|)/\mathrm{d}\omega = 0$ provides the repeated solution that the maximum value occurs when $\omega = \omega_n$. This is denoted $\hat{f}_n(\mathrm{PP}\{Q_v\})$ and provides a basis for determining f_n independent of the spectral phase.

2.2. Real part of the spectra

The real part of G_x is

$$\mathrm{Re}\{G_x(\omega)\} = \frac{1}{m}\frac{\omega_n^2 - \omega^2}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2} = \frac{\omega_n^2 - \omega^2}{mD(\omega)}, \tag{6}$$

and its frequency response is shown in [2,4]. From (6) it is clear that the frequency at which $Re\{G_x(\omega)\} = 0$ is f_n. Applying (3) it is also seen that this is also one of the solutions for both the acceleration and the velocity spectra, thus $\hat{f}_n(\mathrm{Re}\{\cdot\})$ defines f_n exactly for all response parameters. The trivial solution ($\omega = 0$) is ignored as in practice the frequency range analysed seldom extends to dc. This is useful as the acceleration or velocity responses are more commonly measured than displacement, and the conversion to produce the displacement signal (which can be problematic [7]) is thus not required to determine f_n.

2.3. Imaginary part of the Q_v spectrum

The imaginary part of Q_v is

$$\mathrm{Im}\{Q_v(\omega)\} = \frac{2\zeta\omega_n\omega^2}{mD(\omega)}, \tag{7}$$

(where $D(\omega)$ is defined in (6)) and the frequency response is as that shown in [4] for G_x when multiplied by ω. Equating the derivative with respect to ω to zero shows that $\hat{f}_n(\mathrm{Im}\{Q_v\})$ determines f_n exactly. This property is valid for G_v (or V) only. In practice the nearest frequency point is taken; the form of the resonance peak of $\mathrm{Im}\{Q_v\}$ does not make it a good candidate to *accurately* apply curve fitting to obtain greater precision for $\hat{f}_n$.

2.4. Measurement of system damping

The damping, ζ, can be determined from the $\mathrm{Re}\{G_x\}$ spectrum as

$$\zeta = \frac{f_b^2 - f_a^2}{4f_n^2}, \tag{8}$$

where f_a and f_b are frequencies identified by the maximum and minimum amplitude of $\mathrm{Re}\{G_x\}$. Eq. (8) is valid for both low and high values of damping [1–3]. In practice a discrete frequency axis is used and the maximum error in determining a single frequency point is thus $\pm\Delta f/2$. However, the effect of discretization on ζ is more complex. By permuting the error of each frequency in (8) the minimum and maximum values can be found as

$$\left\{ \begin{array}{c} \zeta_{\min} \\ \zeta_{\max} \end{array} \right\} = \frac{(f_b \mp \Delta f/2)^2 - (f_a \pm \Delta f/2)^2}{(f_n \pm \Delta f/2)^2}. \tag{9}$$

$\zeta_{\min}$ can now be expressed as

$$\zeta_{\min} = \frac{\zeta}{(1 + \Delta f/f_n)} - \frac{(f_b + f_a)\Delta f}{4f_n(1 + \Delta f/f_n)}. \tag{10}$$