Accepted Manuscript

Locally robust methods and near-parametric asymptotics

Spiridon Penev, Kanta Naito

PII: S0047-259X(17)30643-7
DOI: https://doi.org/10.1016/j.jmva.2018.06.006
Reference: YJMVA 4374

To appear in: *Journal of Multivariate Analysis*

Received date : 26 October 2017

Locally robust methods and near-parametric asymptotics

Spiridon Penev [a,*], Kanta Naito [b,]

[a]The University of New South Wales, Department of Statistics, Sydney, 2052 NSW, Australia
[b] Division of Mathematical Sciences, Shimane University, Matsue, Japan

Abstract

It has already been shown theoretically and numerically that infusing a little localization in the likelihood-based methods for regression and for density estimation can actually improve the resulting estimators with respect to suitably defined global risk measures. Thus various local likelihood methods have been suggested. In this paper, we demonstrate that a similar effect can also be observed with respect to robust estimation procedures. Localized versions of robust density estimation procedures perform better with respect to global risk measures based on minimization of Bregman divergence measures.

Keywords: Bregman divergence, Kernel, Power divergence, Risk, Robustness
2000 MSC: 62G07, 62E20, 62G35

1. Introduction

Local likelihood estimation procedures were introduced in [13]. Their original application was in regression, especially in generalized regression models. The extension of the application in a density estimation context was proposed in [6, 9]. Paper [6] summarizes the best features of the local likelihood density estimation approach as a truly semiparametric method. “The estimators run the gamut from a fully parametric fit to almost fully nonparametric with only a single smoothing parameter to be chosen”.

This argument is also taken on board in [3]. They include an additional argument about the usefulness of the local likelihood approach in density estimation. Later, paper [12] further expanded the arguments of [3] with a more general bandwidth analysis. In a nutshell, the above mentioned run of the estimators is controlled by the choice of the smoothing parameter h, the bandwidth. With “small h” one is closer to the fully non-parametric fit and with “large h” one is closer to the parametric fit. Depending on how close the true density is to the parametric model, different terms in the expansion of a suitably defined estimation risk may dominate.

In Section 2.1 of [3], the procedure is interpreted as the one that minimizes, at each value x of the argument, the locally weighted Kullback–Leibler divergence between the “true” and the model density. We denote by F the cumulative distribution function and assume that the density f exists. As they say in their introduction, conceptually it is indeed rarely the case that we are sure or are justified to assume that the underlying density f belongs precisely to a parametric model of the form $g(x, \theta)$ with $\theta \in \Theta \subset \mathbb{R}^p$. It is more realistic and reasonable to assume that f belongs to a tubular neighborhood $\cup_{\theta\in\Theta}\{f : D(f, g_\theta) \leq \epsilon\}$. Here $D(\cdot, \cdot)$ denotes some global measure of divergence (proximity) between f and the model. One such reasonable measure could be the Kullback–Leibler divergence between f and $g_\theta = g(\cdot, \theta)$, viz.

$$D(f, g_\theta) = \mathrm{E}_f \left[\ln\{f(X)/g(X, \theta)\}\right].$$

We are interested in cases where ϵ, although being very small, is not zero so that the true parametric model does not hold globally (but may hold locally at each point x with a parameter θ dependent on that point). In that case, it is a good idea to infuse a local adaptation to the global likelihood by considering maximization of an expression of the form

$$\sum_{i=1}^{n} K\left(\frac{X_i - t}{h}\right) \ln g(X_i, \theta) \tag{1}$$

based on data $X_1, \ldots, X_n \sim f$ with $K(z)$ being some suitable nonnegative kernel function symmetric around $z = 0$ with $K(0) = 1$ and h being a bandwidth controlling the extent of localization. In fact, further modifications of the simple

*Corresponding author
Email addresses: s.penev@unsw.edu.au (Spiridon Penev), naito@riko.shimane-u.ac.jp (Kanta Naito)