Journal of Multivariate Analysis 166 (2018) 129–149

Contents lists available at ScienceDirect

Journal of Multivariate Analysis

journal homepage: www.elsevier.com/locate/jmva

Spatial modeling of rainfall accumulated over short periods of time

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ARTICLE INFO

Article history:
Received 14 December 2016

AMS subject classifications:
60G15
60G60
62M30

Keywords:
Gauss–Hermite quadrature
Gaussian random fields
Latent processes
Mixed distributions
Spatial intermittency
Stochastic dominance

ABSTRACT

This article proposes a new random field model to describe the spatial variation of rainfall amounts accumulated over short periods of time. The model is intended to satisfy a set of desiderata motivated by the understanding of rainfall generating mechanisms and exploratory data analysis of datasets of this type. First and second order properties of the proposed model are derived, including the mean and covariance functions, as well as the families of marginal and bivariate distributions. Properties of the proposed model are shown by a mix of analytical derivations and numerical exploration that use Gauss–Hermite quadrature to approximate the required integrals. The proposed model also satisfies a stochastic dominance property, which is argued to be sensible and consistent with most rainfall data of this type. A study of identifiability is carried out, which strongly suggests all model parameters are identifiable. The generalized method of moments is proposed to estimate the parameters, and the properties of these estimators are explored based on simulated data.

1. Introduction

Rainfall is an hydrological variable of paramount importance since it is the main driving input for many other hydrological and meteorological processes. Rainfall is also a key input for the natural evolution of ecosystems and the operation of man-made structures, such as water distribution and irrigation systems. As is true for most earth processes, rainfall is a non-negative variable that displays substantial variation in both space and time. In particular, it displays substantial spatial association, where measurements taken at nearby locations tend to be more similar than measurements taken farther apart. The data we aim at describing in this work are ground-based rainfall measurements, such as those collected by tipping buckets. Since the size of a bucket is negligible compared to the size of the region where the phenomenon is observed, these spatial data are of the geostatistical type [8], as it can be assumed that any location of the region has an associated rainfall amount. A different kind of rainfall measurements are those collected from remote-based instruments, such as radars or satellites, but these will not be considered here.

In this work we consider the spatial modeling of rainfall accumulated over a short period of time. This may be as short as an hour or as long as a few days, with one day being the typical accumulation period. Because of the nature of rainfall processes and the short accumulation period, at any location in the region of interest it may or may not rain during the accumulation period, and either of these events occurs with positive probability. As a result, the marginal distributions of

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https://doi.org/10.1016/j.jmva.2018.02.009
0047-259X/

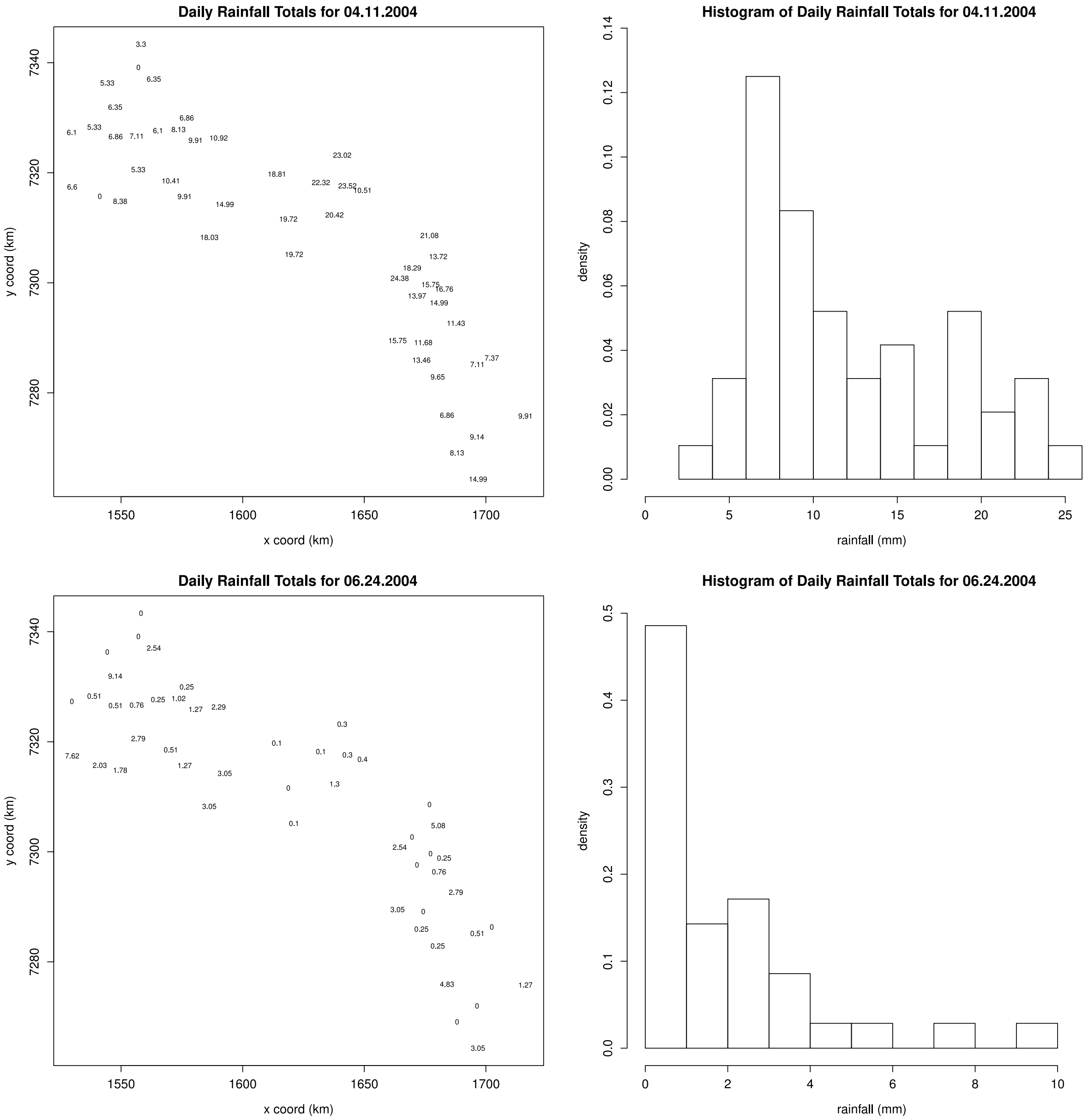

Fig. 1. Left panels: Rainfall amounts collected at 50 locations in 2014 over the Guadalupe River Basin in central Texas (United States) during April 11 and June 24, respectively. Right panels: Histograms of *positive* rainfall amounts for the same two days. Thanks to Hongjie Xie for providing the data.

rainfall are mixtures of a (degenerate) distribution supported at zero and an absolutely continuous distribution supported on the positive real line. Likewise, the joint distributions of rainfall amounts at several locations are mixtures of distributions supported at the origin, lines, planes, etc.; this feature of rainfall is called spatial *intermittency*. At the end of the accumulation period, a rainfall event splits the region of interest into two subregions: one that includes the locations where it did not rain, called the *dry subregion*, and the other that includes the locations where it did rain, called the *wet subregion*. Both subregions are expected to have a high degree of 'connectivity', since two nearby locations tend to belong to the same subregion (dry or wet).

To illustrate the rainfall features mentioned above and motivate other expected distributional features of rainfall to be described below, we show some examples of typical rainfall data we aim to describe. Fig. 1 summarizes daily rainfall measured by tipping buckets at 50 locations over the Guadalupe River Basin in central Texas (USA). The left panels display rainfall amounts collected in 2004 during April 11 and June 24, respectively, while the right panels display the histograms of *positive* rainfall amounts for the same two days. The spatial intermittency is apparent. Fig. 2 displays maps of reflectivity, a