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Higher-order asymptotic theory of shrinkage estimation for general statistical models

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Abstract

In this study, we develop a higher-order asymptotic theory of shrinkage estimation for general statistical models, which includes dependent processes, multivariate models, and regression models (i.e., non-independent and identically distributed models). We introduce a shrinkage estimator of the maximum likelihood estimator (MLE) and compare it with the standard MLE by using the third-order mean squared error. A sufficient condition for the shrinkage estimator to improve the MLE is given in a general setting. Our model is described as a curved statistical model $p(\cdot;\theta(u))$, where θ is a parameter of the larger model and u is a parameter of interest with $\dim u < \dim\theta$. This setting is especially suitable for estimating portfolio coefficients u based on the mean and variance parameters θ. We finally provide the results of our numerical study and discuss an interesting feature of the shrinkage estimator.

Keywords: Curved statistical model, Dependent data, Higher-order asymptotic theory, Maximum likelihood estimation, Portfolio estimation, Regression model, Shrinkage estimator, Stationary process

1. Introduction

Since the seminal works by Stein and James [8, 18], termed JS works hereafter, shrinkage analysis has been extended to estimate a linear regression model $\boldsymbol{X} = \boldsymbol{U}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$. Based on the least squares (LS) estimator $\hat{\boldsymbol{\beta}}_{LS}$ of $\boldsymbol{\beta}$, JS-type estimators have been proposed. When $\boldsymbol{\varepsilon} \sim \mathcal{N}_n(0, \sigma^2 I_n)$, Arnold [2] proposed the estimator

$$\hat{\boldsymbol{\beta}}_S = \left(1 - \frac{c\hat{\sigma}^2}{\|\boldsymbol{U}\hat{\boldsymbol{\beta}}_{LS}\|^2}\right)\hat{\boldsymbol{\beta}}_{LS},$$

where c is a positive constant and $\hat{\sigma}^2 = \|\boldsymbol{X} - \boldsymbol{U}\hat{\boldsymbol{\beta}}_{LS}\|^2/(n-p)$ with $p = \dim\boldsymbol{\beta}$ is an estimator of σ^2, and showed that $\hat{\boldsymbol{\beta}}_S$ improves $\hat{\boldsymbol{\beta}}_{LS}$ with a suitable choice of c if $p \geq 3$.

Although shrinkage estimation has developed in many directions, one stream has become known as empirical Bayes estimation. For instance, by adopting a mixed linear regression model, Lahiri and Rao [12] evaluated the mean squared error (MSE) of an empirical linear Bayes estimator of small area means and discussed MSE estimation. They showed this MSE estimator to be robust with respect to non-normality.

We often use statistical methods designed for independent samples when the observations may in fact be dependent (e.g., financial engineering and econometrics). In such cases, it is important to investigate the influence of the statistical method to be used. For a vector-valued Gaussian stationary process with a mean vector $\boldsymbol{\mu}$, Taniguchi and Hirukawa [20] studied the MSE of the sample mean $\hat{\boldsymbol{\mu}}$ and the JS estimator $\hat{\boldsymbol{\mu}}_{JS}$ of $\boldsymbol{\mu}$. Then, they provided a set of sufficient conditions for $\hat{\boldsymbol{\mu}}_{JS}$ to improve $\hat{\boldsymbol{\mu}}$ in terms of the spectral density matrix. For a time series regression model, Senda and Taniguchi [16] evaluated the MSE of the JS estimator of regression coefficients and compared it with that of the LS estimator. Then, the authors provided sufficient conditions for the JS estimator to improve the LS estimator. For the autocovariance functions of a Gaussian stationary process, Taniguchi et al. [22] introduced a modified autocovariance estimator and then compared its MSE with that of the usual sample autocovariance estimator.

From the statistical asymptotic theory perspective, to compare the improvement of shrinkage estimators with that of typical estimators, a third-order asymptotic theory is required. For an iid curved exponential family with the parameter u, Amari [1] developed the third-order asymptotic theory of estimation and testing for u. For a more general setting that includes not only iid observations but also regression, multivariate, and time series models, Taniguchi and Watanabe [23] developed the third-order asymptotic theory of inference for the family of curved probability densities

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