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Multivariate generalized Pareto distributions: Parametrizations, representations, and properties

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Abstract

Multivariate generalized Pareto distributions arise as the limit distributions of exceedances over multivariate thresholds of random vectors in the domain of attraction of a max-stable distribution. These distributions can be parametrized and represented in a number of different ways. Moreover, generalized Pareto distributions enjoy a number of interesting stability properties. An overview of the main features of such distributions is given, expressed compactly in several parametrizations, giving the potential user of these distributions a convenient catalogue of ways to handle and work with generalized Pareto distributions.

Keywords: Exceedances, maxima, stable tail dependence function, tail copula, linear combination.

1. Introduction

A core theme in univariate extreme-value analysis is to fit a generalized Pareto (GP) distribution to a sample of excesses over a high threshold. Since univariate GP distributions can be described in terms of a scale parameter and a shape parameter, statistical inference using frequentist or Bayesian likelihood techniques is straightforward, at least for values of the shape parameter at which the Fisher information matrix is finite.

For multivariate extremes, matters are more complicated. First, there is no universal definition of an exceedance of a multivariate threshold. Second, whatever the definition that is selected, the family of distributions proposed by asymptotic theory is no longer parametric.

Following Rootzén and Tajvidi [17], we say that a sample point $\boldsymbol{y} \in \mathbb{R}^d$ exceeds a multivariate threshold $\boldsymbol{u} \in \mathbb{R}^d$ as soon as one of its coordinates exceeds the corresponding threshold coordinate, i.e., $y_j > u_j$ for at least one $j \in \{1, \ldots, d\}$. In dimension $d = 2$, the shape of the excess region $\{\boldsymbol{y} \in \mathbb{R}^d : \boldsymbol{y} \nleq \boldsymbol{u}\}$ is that of the letter L upside-down; here and in what follows, inequalities between vectors are meant component-wise. The excess region covers a larger part of the sample space than the one for most other threshold exceedance definitions, for instance, that $\boldsymbol{y}$ exceeds $\boldsymbol{u}$ when $\boldsymbol{y} > \boldsymbol{u}$, i.e., $y_j > u_j$ for all $j \in \{1, \ldots, d\}$.

The class of GP distributions that arises from the first definition of a multivariate exceedance is derived directly from the family of multivariate generalized extreme-valued (GEV) or max-stable distributions; see, e.g., Beirlant et al. [2, Section 8.3] or Rootzén and Tajvidi [17]. Still, such multivariate GP distributions have enjoyed much less popularity than their univariate counterparts. One reason may be that the multivariate versions are mathematically more involved. Their support is (a subset of) $\{\boldsymbol{x} : \boldsymbol{x} \nleq \boldsymbol{0}\}$, the complement of the negative orthant, where $\boldsymbol{x} = \boldsymbol{y} - \boldsymbol{u}$ represents the excess vector, at least one coordinate of which is positive by definition. The unusual shape of the support introduces a nontrivial dependence structure uncommon to other families of multivariate distributions.

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