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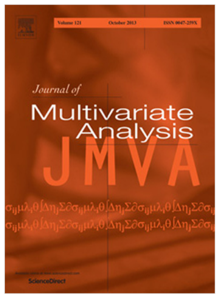

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ACCEPTED MANUSCRIPT

On the joint tail behavior of randomly weighted sums of heavy-tailed random variables

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Abstract

We focus on the joint tail behavior of randomly weighted sums $S_n = U_1X_1+\cdots+U_nX_n$ and $T_m = V_1Y_1+\cdots+V_mY_m$. The vectors of primary random variables (X_1, Y_1), (X_2, Y_2), ... are assumed to be independent with dominatedly varying marginal distributions, and the dependence within each pair (X_i, Y_i) satisfies a condition called strong asymptotic independence. The random weights U_1, V_1, ... are non-negative and arbitrarily dependent, but they are independent of the primary random variables. Under suitable conditions, we obtain asymptotic expansions for the joint tails of (S_n, T_m) with fixed positive integers n and m, and (S_N, T_M) with integer-valued random variables N and M that are independent of the primary random variables. When the marginal distributions of the primary random variables are extended regularly varying, the result is proved to hold uniformly for any n and m under stronger conditions. Our results rely critically on moment conditions that are generally easy to check.

Keywords: Asymptotics, dominated variation, joint tail behavior, randomly weighted sum, regular variation, strong asymptotic independence.

1. Introduction

Let $\{(X_i, Y_i) : i \in \mathbb{N}\}$ be a sequence of mutually independent random vectors, and let $\{U_i, V_i : i \in \mathbb{N}\}$ be a sequence of arbitrarily dependent non-negative random variables. Define

$$S_n = \sum_{i=1}^{n} U_iX_i \quad \text{and} \quad T_m = \sum_{j=1}^{m} V_jY_j,$$

where n and m can either be fixed positive integers, infinite, or random variables taking values on non-negative integers. In this paper, we are concerned with asymptotic expansions for $\Pr(S_n > x, T_m > y)$ as $(x, y) \to (\infty, \infty)$.

The asymptotic tail behavior of a single sum S_n has been extensively studied when the primary random variables X_is have heavy-tailed distributions; see, e.g., [5, 8, 10, 21, 24]. Interest in this problem stems from the flexibility of the random weights $U_1, U_2, \ldots$, which makes S_n adaptable to various contexts. For example, in risk theory X_i is usually interpreted as the net loss of an insurer in period i, and U_i is the accumulated discount factor until time i. Then S_n describes the present value of the total risk over the first n periods, and the study of ruin probabilities, risk measures, etc. boils down to the study of the tail behavior of S_n; see, e.g., [14, 17–20, 23] for some details.

In contrast, there is a dearth of asymptotic results for the joint tail $\Pr(S_n > x, T_m > y)$. However, such an extension is not only meaningful from a theoretical perspective, but also necessary for practical applications. For example, it is virtually impossible today for an insurer to operate with a single line of business. Frameworks have thus been proposed to model surplus processes for multiple lines. Results in the bivariate case can provide invaluable insights into these issues.

We introduce between each pair (X_i, Y_i) of primary random variables a dependence structure referred to as strong asymptotic independence (see Definition 1 below), which is expressed in terms of the marginal and joint tails of

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