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Extreme-value limit of the convolution of exponential and multivariate normal distributions: Link to the Hüsler–Reiß distribution

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Abstract

The multivariate Hüsler–Reiß copula is obtained as a direct extreme-value limit from the convolution of a multivariate normal random vector and an exponential random variable multiplied by a vector of constants. It is shown how the set of Hüsler–Reiß parameters can be mapped to the parameters of this convolution model. Assuming there are no singular components in the Hüsler–Reiß copula, the convolution model leads to exact and approximate simulation methods. An application of simulation is to check if the Hüsler–Reiß copula with different parsimonious dependence structures provides adequate fit to some data consisting of multivariate extremes.

Keywords: Copula, extreme-value limit, parametric bootstrap, parsimonious dependence.

1. Introduction

It is well known (see, e.g., [30]) that scaled vectors of component-wise maxima from the multivariate normal distributions are asymptotically independent, i.e., the limit in distribution is a product of univariate Gumbel or extreme-value distributions. Hüsler and Reiß [16] obtained a limiting non-trivial extreme-value distribution from scaled vectors of component-wise maxima from the multivariate normal by letting the correlation parameters increase to 1 at appropriate rates as the sample size $n \to \infty$. Nikoloulopoulos et al. [27] derived the multivariate extreme-value t copula (t-EV) from the extreme-value limit of the multivariate t distribution with degree of freedom parameter $\nu > 0$ and correlation matrix $\boldsymbol{R} = (\rho_{ij})$. By letting $\rho_{ij} \to 1$ and $\nu \to \infty$ at appropriate rates, the Hüsler–Reiß (HR) distribution was obtained in a simpler form.

Both of these limits are not direct extreme-value limits so that their constructions cannot be used to simulate from the Hüsler–Reiß distribution by approximation with the component-wise maxima of N independent and identically distributed (iid) random vectors for large N. It is however useful to be able to simulate, at least approximately, from the HR distribution. One example of use is to provide inferences of a fitted model by means of a parametric bootstrap.

In this paper, we show how the HR distribution can be obtained as a direct extreme-value limit from the convolution of a multivariate normal vector and an exponential random variable multiplied by a positive vector of constants. The convolution model is studied for spatial data with non-Gaussian dependence and tails in [21].

The derivation of the extreme-value limit of the convolution model is given in Section 2. In Sections 3 and 4, we study how the parameters of the HR distribution can be mapped to the parameters of the convolution model, starting with the trivariate case. Also, we derive inequalities for the parameters of the HR distribution based on partial correlations. In Section 5, we give details on simulating random variates from the HR distribution and show an example where a parametric bootstrap is used for assessing the adequacy of fit of some fitted HR distributions with parsimonious dependence structures. Section 6 concludes with some discussion.

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