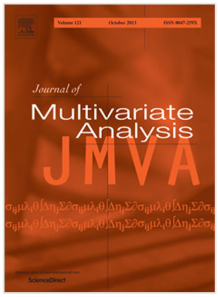
Accepted Manuscript



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PII: S0047-259X(16)00034-8
DOI: http://dx.doi.org/10.1016/j.jmva.2016.02.012
Reference: YJMVA 4091

To appear in: *Journal of Multivariate Analysis*

Received date: 31 May 2015

Please cite this article as: H. Tsukuma, Estimation of a high-dimensional covariance matrix with the Stein loss, *Journal of Multivariate Analysis* (2016), http://dx.doi.org/10.1016/j.jmva.2016.02.012

Estimation of a high-dimensional covariance matrix with the Stein loss

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February 15, 2016

Abstract

The problem of estimating a normal covariance matrix is considered from a decision-theoretic point of view, where the dimension of the covariance matrix is larger than the sample size. This paper addresses not only the nonsingular case but also the singular case in terms of the covariance matrix. Based on James and Stein's minimax estimator and on an orthogonally invariant estimator, some classes of estimators are unifiedly defined for any possible ordering on the dimension, the sample size and the rank of the covariance matrix. Unified dominance results on such classes are provided under a Stein-type entropy loss. The unified dominance results are applied to improving on an empirical Bayes estimator of a high-dimensional covariance matrix.

AMS 2010 subject classifications: Primary 62H12; secondary 62C12.

Key words and phrases: Empirical Bayes method, inadmissibility, Moore-Penrose pseudo-inverse, portfolio selection, pseudo Wishart distribution, singular multivariate normal distribution, singular Wishart distribution, statistical decision theory.

1 Introduction

This paper addresses the problem of a normal covariance matrix relative to the Stein loss, where the dimension of the covariance is larger than the sample size. This problem is precisely formulated as follows: Let $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$ be independently and identically distributed as $\mathcal{N}_p(\mathbf{0}_p, \boldsymbol{\Sigma})$. Assume that $p > n$ and $\boldsymbol{\Sigma}$ is a $p \times p$ positive definite matrix of unknown parameters. Denote $\boldsymbol{S} = \sum_{i=1}^{n} \boldsymbol{X}_i \boldsymbol{X}_i^\top$. Then $\boldsymbol{S}$ is distributed as

$$\boldsymbol{S} \sim \mathcal{W}_p(n, \boldsymbol{\Sigma}). \tag{1.1}$$

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