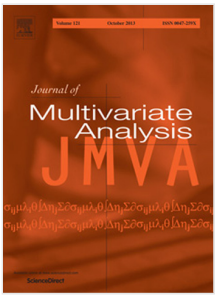

A general setting for symmetric distributions and their relationship to general distributions

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A general setting for symmetric distributions and their relationship to general distributions

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Abstract

A standard method of obtaining non-symmetrical distributions is that of modulating symmetrical distributions by multiplying the densities by a perturbation factor. This has been considered mainly for central symmetry of a Euclidean space in the origin. This paper enlarges the concept of modulation to the general setting of symmetry under the action of a compact topological group on the sample space. The main structural result relates the density of an arbitrary distribution to the density of the corresponding symmetrised distribution. Some general methods for constructing modulating functions are considered. The effect that transformations of the sample space have on symmetry of distributions is investigated. The results are illustrated by general examples, many of them in the setting of directional statistics.

Keywords: Directional statistics, Skew-symmetric distribution, Symmetry-modulated distribution, Transformation
2010 MSC: 62E10, 62H05

1. Introduction

Because many appealing distributions are symmetrical but many data sets are not, intense work has been dedicated to the study of families of tractable distributions obtained by modifying standard symmetrical distributions such

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