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Maximum of the sum of consecutive terms in random permutations

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Abstract

We determine the limit distribution of the maximum of the sum of a fixed number of the consecutive terms in a random permutation of the first n positive integers as n tends to infinity.

Keywords: Random permutations, sum of consecutive terms, extreme values.
2010 MSC: Primary 60C05; Secondary 60G70.

1. Introduction and main result

Random permutations have been the subject of a number of research papers and many interesting results were obtained. See, for example, Chapter 6 of the book [1] and the references therein. There are also new and exciting applications in biology (see, for example, [2, 3]).

In this paper we investigate the asymptotic behavior of the maximum of the sum of a fixed number of the consecutive terms in a random permutation of the first n positive integers.

Let us start with the notation. Let Ω_n be the set of all permutations of the set $\mathbb{N}_n = \{1, 2, \ldots, n\}$. Suppose that probability of each permutation $\omega \in \Omega_n$ is $1/n!$. For any $\omega = (a_1, a_2, \ldots, a_n)$ let us denote

$$X_{nj}(\omega) = a_j + a_{j+1} + \cdots + a_{j+k-1}, \quad j \in \mathbb{N}_n, \ \ k < n,$$

where $a_{n+j} = a_j$ for $j = 1, 2, \ldots$ and

$$M_n = \max\{X_{n1}, \ldots, X_{nn}\}.$$

Then, $X_{n1}, \ldots, X_{nn}$ is a sequence of dependent random variables, satisfying the condition of strict stationarity. For any fixed $k \in \mathbb{N}$ we determine the limiting distribution of the random variable M_n as $n \to \infty$.

Note that the general results related to extreme values in stationary sequences were obtained in [4], see also [5]. We shall also use certain facts and techniques from the theory of partitions, see [6, 7]. A problem similar to the one defined above, but not related to the theory of partitions, is considered by [8].

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