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On mixture autoregressive conditional heteroskedasticity

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ABSTRACT

We consider mixture univariate autoregressive conditional heteroskedastic models, both with Gaussian or Student t-distributions, which were proposed in the literature for modeling nonlinear time series. We derive sufficient conditions for second order stationarity of these processes. Then we propose an algorithm in matrix form for the estimation of model parameters, and derive a formula in closed form for the asymptotic Fisher information matrix. Our results are proved by using the theory of time series models with Markov changes in regime. An illustrative example of the theoretical results and a real application on financial data complete the paper.

1. Introduction

Wong and Li (2001) introduced mixture autoregressive conditional heteroskedastic models (with Gaussian distribution), denoted in short by MAR–ARCH. These models consist of a mixture of M autoregressive components with autoregressive conditional heteroskedasticity; that is, the conditional mean of the observable variable y_t follows an AR process, and the conditional variance of y_t follows an ARCH process in the sense of Engle (1982). The above authors have emphasized that MAR–ARCH models are powerful tools for modeling nonlinear time series. Several papers illustrate the usefulness of such models to capture conditional heteroskedasticity and other features of financial time series. See, for example, Wong and Li (2000, 2001), Fong and Wong (2008), Chan et al. (2009), and Zhu et al. (2010). Here we briefly summarize some peculiarities of the considered models. Firstly, it is possible that a mixture of a nonstationary MAR component and a stationary MAR component results in a stationary process. Further, it is possible to mix both explosive and inexplosive ARCH components and yet the series is still second order stationary. Secondly, conditional distributions of the time series given the past history under these models are changing over time. As a consequence, such distributions can be multimodal. Furthermore, the conditional expectation of the observable variable may not be the best predictor of the future values. Thirdly, another important feature of the MAR–ARCH model is the ability to model changing conditional variance.

A Student t-mixture autoregression without mixture ARCH errors was studied by Wong et al. (2009). The use of t-distributed errors in each component of the model allows conditional leptokurtic distributions that account for the commonly observed excess unconditional kurtosis in financial data. Some numerical applications to heavy-tailed financial data have been illustrated in Wong et al. (2009).

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Stationarity conditions and (squared) autocorrelation functions were derived in Wong and Li (2001) only for special classes of MAR–ARCH models. As stated by those authors, the derivation of second order stationarity conditions for the general MAR–ARCH model is rather complicated, because it involves the autocovariance terms and the unconditional expectation of the residuals. More recently, Saikkonen (2007) has given stationarity conditions in a general setting and proved the existence of second order moments for mixtures of linear vector autoregressive models with a conventional ARCH term, by using the stability theory of Markov chains. Francq and Zakoïan (2001) also provide stationarity conditions for Markov switching VARMA models, and matrix expressions for higher order moments of these processes are established in Cavicchioli (2017). Recent developments on the stationarity of mixture vector autoregressive models can be found in Cavicchioli (2016).

Wong and Li (2001) discussed the parameter estimation of a MAR–ARCH model (with Gaussian distribution) by using the EM algorithm. Their iterative procedure for maximizing the log likelihood function of the model consists of two steps, called the *expectation* step (E-step) and the *maximization* step (M-step). Then the estimates of model parameters are obtained by iterating these computational steps until convergence. This procedure was successively repeated in Wong et al. (2009) for Student t-mixture autoregressive models without mixture ARCH errors. Other methods for the estimation of general mixed models can be found in Sallas and Harville (1981), Breslow and Clayton (1993), and Chauveau (1995). More recent results on the estimation of the parameters for mixture autoregressions can be found in Aknouche (2013). This author proposes a recursive online EM (REM) algorithm which is based on an unobserved weighted least squares criterion. Under the normality assumption on the component distributions, such a criterion is asymptotically equivalent to the complete-data likelihood. The unobserved weights are estimated from data using approximations of the predictive probabilities instead of the filtered ones as is usual for the EM algorithm. It is shown in Aknouche (2013) that the EM and REM algorithms provide roughly similar estimates, especially for moderate and long time series.

The main contributions of the present paper are as follows. First, we complete and extend in a very general setting the second order stationarity conditions of MAR–ARCH models, which are discussed in Wong and Li (2001) only for particular cases. Then we compute explicitly the expectation and the second order moment of such models.

Second, we propose an algorithm to estimate the parameters of a MAR–ARCH model (in the both cases Gaussian and Student t-distributions), which is alternative to that developed by Wong and Li (2001) at the scalar-level. The new proposed estimation procedure can be considered as an extended version of the one of Hamilton (1994, Section 21) from AR–ARCH models to the MAR–ARCH models case. Our approach is different from the one used by Wong and Li (2001), and confirms the validity of their results by suitable grouping the element-by-element derivatives in a vector. However, our recursion equations, written as concisely as possible at the vector–matrix level, improve computational performance since they are readily programmable and in addition greatly reduce the computational cost. The proposed algorithm is first described for estimating the parameters of a MAR–ARCH model with Gaussian errors, and then it is extended to the case of Student t-distribution. Our results also include those obtained in Wong et al. (2009) for the subclass of Student t-mixture autoregressive models without mixture ARCH errors.

Third, a further advantage of the proposed approach is that, in both situations, an expression in closed form for the observed Fisher information matrix can be provided explicitly by using appropriate techniques from matrix differential calculus. The matrix formula we obtain is potentially useful for statistical inference of MAR–ARCH models. Despite the increasing literature concerning Markov switching models, explicit expression of the limiting covariance matrix of the maximum likelihood estimator (MLE) has not been given at the present. Certain authors like Bickel et al. (1998) have studied asymptotic properties of the MLE for Markov switching VAR models, but the expression of their corresponding limiting variance has a rather complicated form. Wong and Li (2001) have emphasized the difficulty of obtaining limiting variance of the MLE in the framework of the EM algorithm. The results given in the present paper solve this problem for MAR models with mixture ARCH errors.

The usefulness of MAR–ARCH models for modeling financial time series was largely illustrated in the quoted papers with examples and numerical applications to real data. Here we provide a further real application based on financial data and some examples which illustrate our theoretical results.

The paper is organized as follows. Section 2 provides the description of the MAR–ARCH model whose autoregressive components are conditionally Gaussian. Then we derive sufficient conditions for second order stationarity of the general MAR–ARCH model. The estimation procedure in matrix form for such models is described in Section 3. We derive the tth m-score in a matrix form and obtain the second derivatives of the log likelihood with respect to the parameters in the mth component approximately in a matrix closed form. To find maximum likelihood (ML) estimates of model parameters we employ the Newton–Raphson algorithm, based on the obtained matrix formulas. Section 4 is devoted to present a closed form expression of the asymptotic Fisher information matrix for MAR–ARCH models with Gaussian errors. In Section 5 we study MAR–ARCH models obtained by replacing the Gaussian assumption with the Student t-distribution. These models generalize those considered in Wong et al. (2009). For such models, we derive the log likelihood function and obtain expressions in closed form for the score and Hessian functions in the same manner as the Gaussian case. These formulas are then used in the estimation step to drive numerical iterations. Section 6 is devoted to illustrate the usefulness of the considered model via a real example using the daily Standard and Poor composite 500 stock price index in the period from 2000 to 2015. Finally, Section 7 concludes. Proofs are given in the Appendix.

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