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Least squares estimation for the drift parameters in the sub-fractional Vasicek processes

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ABSTRACT

While the statistical inference of Vasicek processes driven by both Brownian motions and fractional Brownian motions has a long history, the statistical analysis for the Vasicek model driven by other fractional Gaussian processes is obviously more recent. This paper considers the parameter estimation problem for Vasicek processes driven by sub-fractional Brownian motions with the known Hurst parameter greater than one half. Since the maximum likelihood estimators are hardly analyzed because of the stochastic integrals with singular kernels, least squares estimators for drift parameters are provided based on time-continuous observations. The strong consistency results as well as the asymptotic distributions of these estimators are obtained in both the non-ergodic case and the null recurrent case.

1. Introduction

Since the seminal work of Vasicek (1977), the Vasicek process driven by standard Brownian motion has been extensively applied in various fields, as diverse as economics and finance, biology, physics, chemistry, medicine and environmental studies. Indeed, when the Vasicek model is used to describe some phenomena, it is important to identify the unknown parameters in this model. As a result, the parameter estimation problem for the Vasicek process driven by Brownian motion has played an important role in econometrics and becomes an interesting problem in the literature. The popular method is the maximum likelihood estimation based on the Girsanov density (Liptser and Shiryaev, 2001; Dietz and Kutoyants, 2003). Moreover, the asymptotic properties and bias corrections for the maximum likelihood estimators are extensively studied in Tang and Chen (2009), Wang et al. (2011), Yu (2012) and Zhou and Yu (2015). The least squares estimators can be found in Kasonga (1988) and the nonparametric method has been proposed by Fan (2005). For a more comprehensive discussion, we refer to Prakasa Rao and Kendall (1999) and Kutoyants (2004) and the references therein.

In the past decade, many empirical studies have shown that long memory is a common occurrence in macroeconomic and financial data (see, for example, Lo (1991); Comte and Renault (1996, 1998); Andersen et al. (2003); Granger and Hyung (2004); Xiao et al. (2011); Comte et al. (2012); Chronopoulou and Viens (2012a); Chronopoulou and Viens (2012b); Liu et al. (2013); Corlay et al. (2014)). Moreover, over the past few years, long-range dependence stochastic processes have been intensively used as models for various scientific areas, such as econometrics, hydrology, telecommunication, turbulence,

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image processing, finance and so on. Actually, the best known and widely used stochastic process that exhibits long-range dependence is of course the fractional Brownian motion (fBm), which is a suitable generalization of the standard Brownian motion. Particularly, the fBm produces burstiness, self-similarity and long-range dependence and has stationary increments in the sample path behavior, which are the important behavior of financial time series. Consequently, the fBm is a usual candidate to capture financial phenomena of long-range dependence. Some surveys and complete literatures could be found in Nualart (2006), Biagini et al. (2008), and Mishura (2008). The development of the application for fBm naturally led to the statistical inference for stochastic models driven by this process. Consequently, the parameter estimation problem for stochastic models driven by fBm has been of great interest in the past decade, and besides being a challenging theoretical problem. Some earlier works include classic parametric methods, such as maximum likelihood estimation (see, for example, Kleptsyna and Le Breton (2002); Tudor and Viens (2007); Brouste (2010); Tanaka (2013); Kozachenko et al. (2015)), least squares approach (see, for instance, Hu and Nualart (2010); Belfadli et al. (2011); Tanaka (2015); Azmoodeh and Viitasaari (2015)), method of moments (see, for instants, Es-Sebaiy and Viens (2015); Barboza and Viens (2017)), bayesian inference (see, for example, Lysy and Pillai (2013); Beskos et al. (2015)), nonparametric method (see, for example, Mishra and Prakasa Rao (2011a, 2011b); Saussereau (2014)) and so on. An extensive review on most of the recent developments related to the parametric and other inference procedures for stochastic models driven by fBm can be found in Mishura (2008) and Prakasa Rao (2010).

Although fBm has been applied in various scientific areas, many authors have proposed to use some more general fractional Gaussian processes, such as sub-fractional Brownian motion (sfBm), bi-fractional Brownian motion and weighted-fractional Brownian motion. However, in contrast to the extensive studies on fBm, there has been only a little systematic investigation on the statistical inference of other fractional Gaussian processes. The main reason for this is the complexity of dependence structures fractional Gaussian processes which do not have stationary increments. Diedhiou et al. (2011) propose the maximum likelihood estimator for the mean reversion speed parameter in the sub-fractional Ornstein–Ulhenbeck processes (SFOUP) by the Girsanov transform. Kuang and Xie (2015) provide the maximum likelihood estimator of the drift parameter in a simple linear model driven by sub-fractional Brownian motion using a random walk approximation method. Kuang and Liu (2015) investigate the maximum likelihood estimators for the drift sub-fractional Brownian motion and study the strong consistency, asymptotic distributions and the Berry–Esséen bounds for these estimators. Shen et al. (2016) study the least squares estimator (LSE) of drift coefficient for the Ornstein–Uhlenbeck process driven by a weighted fractional Brownian motion. El Machkouri et al. (2016) consider the least squares estimator for the non-ergodic Ornstein–Uhlenbeck process with some special Gaussian processes. Mendy (2013) considers the problem of estimating the mean reversion speed for SFOUP. Although the mean reversion speed parameter in SFOUP was considered (Mendy, 2013; El Machkouri et al., 2016), in the "Real-world", the long term mean is always unknown. Thus, it is important to estimate both the long term mean level and the mean reversion speed in the Vasicek process driven by sfBm. In this paper, using the least squares method, we consider the problem of estimating both the long term mean level and the mean reversion speed for sub-fractional Vasicek process (SFVP) based on continuous observation. Moreover, both the strong consistency and the asymptotic distributions are established for LSE based on the technics inspired from Belfadli et al. (2011), Mendy (2013) and El Machkouri et al. (2016).

Our paper is organized as follows. Section 2 contains some preliminaries about sfBm and provides LSE for SFVP. The strong consistency and the asymptotic distributions of LSE in non-ergodic case are established in Section 3. Section 4 is devoted to presenting the asymptotic behaviors of LSE for SFVP in the null recurrent case. Section 5 contains some concluding remarks and directions of further works.

2. Preliminaries and least squares estimators

In this section, we first describe some basic facts on the sfBm and stochastic calculus with respect to sfBm. For more complete presentation on the subject, see Bojdecki et al. (2004), Nualart (2006), Tudor (2007), Bojdecki et al. (2007), Yan and Shen (2010) and Shen and Yan (2014) and the references therein. Then, in the latter part of this section, we present the LSE for both the long term mean level and the mean reversion speed in SFVP.

The sfBm arises from occupation time fluctuations of branching particle systems with Poisson initial condition. It is well-known that sfBm has properties analogous to those of fBm (self-similarity, long-range dependence, Hölder paths). However, in comparison with fBm, sfBm has nonstationary increments. The increments over non overlapping intervals are more weakly correlated and their covariance decays polynomially at a higher rate in comparison with fBm (for this reason, in Bojdecki et al. (2004) it is called sfBm). It is worth to emphasize that the properties mentioned here make sfBm a possible candidate for models which involve long-range dependence, self-similarity and non-stationary. Therefore, sfBm has been used to capture the price fluctuations of the financial asset (see, for example, Liu et al. (2010)).

The so-called sfBm with index $H \in (0, 1)$ is a mean zero Gaussian process $S^H = \{S_t^H, t \geq 0\}$ with $S_0^H = 0$ and the covariance

$$\mathbb{E}\left(S_t^H S_s^H\right) = t^{2H} + s^{2H} - \frac{1}{2}\left[(t+s)^{2H} - |t-s|^{2H}\right], \tag{2.1}$$

for every $s, t \geq 0$. For $H = 1/2$, S^H coincides with the standard Brownian motion. In fact, S^H is neither a semimartingale nor a Markov process unless $H = 1/2$. It is well known that sfBm has the following self-similarity property (see, for example Bojdecki et al. (2004))